We examine the observational appearance of an optically thick, spherically symmetric, relativistic wind (a black hole wind), focusing our attention on the emerging spectrum. In a relativistic flow, the apparent optical depth becomes small (large) in the downstream (upstream) direction due to the Lorentz-Fitzgerald contraction. As a result, the location of the apparent photosphere of the wind is remarkably modified, and there appears the relativistic limb-darkening (center-brightening) effect, where the comoving temperature distribution of the apparent photosphere is enhanced (reduced) at the center (in the limb). In addition, due to the Doppler boost, the observed temperature distribution is greatly changed. These relativistic effects modify the expected spectrum. When the wind speed is subrelativistic, the observed temperature distribution is almost uniform, and the spectra of the black hole wind are blackbody-like. When the wind speed becomes relativistic, on the other hand, the observed temperature distribution exhibits the power-law nature of $\tau_{\text{ph}} \propto r^{-\alpha}$, where $r$ is the distance from the disk center, and the observed spectra become a modified blackbody, which has a power-law part of $\propto \nu^{-\alpha}$, where $\nu$ is the frequency.

### 1. Wind Model

- **steady**
- **spherical**
- velocity $v(R) = c\beta(R)$
- black hole mass $M = 10M_\odot$
- constant mass loss rate
- $\dot{m}\equiv (\text{normalized by } L_\text{E}/c^2)$
- constant comoving luminosity $\dot{d}\equiv (\text{normalized by } L_\text{E}) = 1$

### 2. Relativistic Optical Depth

Due to the Lorentz-Fitzgerald contraction, the relativistic optical depth in the fixed frame is modified. We define an “apparent photosphere” of the optically thick wind as the surface, where the optical depth measured from an observer at infinity becomes unity. The observed temperature is also changed.

$$\tau_{\text{ph}} = \int_{r_{\text{ph}}}^{\infty} \frac{(1 - \cos \theta)\kappa_{0}\rho_{0}ds}{L_{\text{E}} \gamma(1 - \beta \cos \theta)} = 1$$

$$T_{\text{obs}} = \frac{1}{1 + z_{0} - T_{0}} = \frac{1}{1 + \frac{\gamma(1 - \beta \cos \theta)}{\gamma(1 - \beta \cos \theta)}}T_{0}$$

### 3.1 Constant-Speed Wind

**LEFT:** Shape of the apparent photosphere. In the highly relativistic case the apparent photosphere becomes a concave shape for an observer in any direction.

**MIDDLE:** Temperature distribution at the apparent photosphere as a function of radius $r$. The dashed curves are the comoving temperatures, while the solid ones are the observed temperatures.

**RIGHT:** Expected spectra of black hole winds. The dashed curves are the comoving spectra, while the solid ones are the observed spectra.

$\dot{m}\equiv 1000$, $v/c = 0.1, 0.5, 0.7, 0.9$, and $0.99$.  
$\dot{m}\equiv 100$, $v/c = 0.1, 0.5, 0.7, 0.9$, and $0.99$.  
$\dot{m}\equiv 10$, $v/c = 0.1, 0.5, 0.7, 0.9$, and $0.99$.  
$\dot{m}\equiv 1$, $v/c = 0.1, 0.5, 0.7, 0.9$, and $0.99$.

### 3.2 Accelerating Wind

**LEFT:** Shape of the apparent photosphere for the accelerating (thick curves) and constant cases (thin curves).

**MIDDLE:** Temperature distribution of the accelerating (thick curves) and constant cases (thin curves).

**RIGHT:** Expected spectra of black hole winds for the accelerating (thick curves) and constant cases (thin curves).

$\dot{m}\equiv 1000$, $v_{\text{terminal}}/c = 0.1, 0.5, 0.7, 0.9$, and $0.99$.

### 4. Concluding Remarks

In this paper we have assumed the gray atmosphere, where the opacity does not depend on frequency. Strictly speaking, the temperature should be evaluated not at the last scattering surface, but from the surface, where the effective optical depth equals to unity. In a high temperature plasma, the effective optical depth is often smaller than the total optical depth, and the gas becomes dominating. In such a scattering dominated plasma, the emergent spectrum is not a simple blackbody but a modified blackbody. We should consider such a scattering effect.

In addition, in the present study we did not consider the general relativistic effect. Since the present black hole wind is optically thick, the general relativistic effect can be safely ignored in many cases. However, when the mass-loss rate is sufficiently small and/or the wind speed is very close to the speed of light, the apparent photosphere at the center shrinks down to the order of the Schwarzschild radius, and the general relativistic effect becomes important.

### Expected spectra of for several cases.

$\dot{m}\equiv 1000$ (thin curves), $100$ (thick curves), $10$ (very thick curves), and $1$ (very very thick curves).

The dashed curves are the $T_{\text{pti}}$ and $T_{\text{obs}}$.

The expected spectrum. When the wind speed is subrelativistic, the observed temperature distribution is almost uniform, and the spectra of the black hole wind are blackbody-like. When the wind speed becomes relativistic, on the other hand, the observed temperature distribution exhibits the power-law nature of $\tau_{\text{ph}} \propto r^{-\alpha}$, where $r$ is the distance from the disk center, and the observed spectra become a modified blackbody, which has a power-law part of $\propto \nu^{-\alpha}$, where $\nu$ is the frequency.

In the highly relativistic limit of $\gamma \rightarrow 1$, the effective emitting region concentation in the very center of $r = 0$, while $\tau_{\text{ph}}$ is finite. In the case we expand $10^{-3}$ under the condition of $\gamma > 3$, we obtain the approximate relation in $10^{-3}$.  

$$\tau_{\text{ph}} \propto \frac{(1 - \beta \cos \theta)}{\gamma(1 - \beta \cos \theta)}$$

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