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# Where is a Marginally Stable Orbit in Luminous Accretion Disk

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# Abstract


- Impressed by a widespread misunderstanding of the issue, we return to the old question concerning the location of the inner edge of the accretion disk around a black hole. That is, "the inner edge does not coincide with the location of the innermost stable Keplerian circular orbit".
- We showed that the flow does not have a potential minimum for accretion rates, dM/dt >10$L_E/c^2$ (with $L_E$ being the Eddington luminosity and $c$ being the speed of light). This property is realized even for relatively small viscosity parameters (i.e., $\alpha \sim 0.01$), because of the effect of the pressure gradient.
- In conclusion, an argument based on the last circular orbit of a test particle cannot give a correct inner boundary of the super-critical flow, and the inner edge should be determined in connection with radiation efficiency.

# Introduction: Effective potential

Let us consider the circular orbit of a test particle around a non-rotating black hole. A general-relativistic description of the effective potential, $\psi$ eff, leads to

$$\psi_{eff}^{GR}(r) = -\frac{GM}{r}\left(1 + \frac{l_*^2}{c^2 r^2}\right) + \frac{l_*^2}{2r^2}$$

See, e.g., Shapiro, Teukolsky 1983

① first term: gravitational potential (<0)

② factor of first term: relativistic correction（$\propto r^{-3}$）

③ second term: potential due to centrifugal force (>0)

（$l_*$: specific angular momentum of a test particle, $c$ is the speed of light）

The minimum value of the equilibrium ($d\psi / dr = 0$) radius is 3 $r_g$.

For simplicity, we adopted a pseudo-Newtonian potential (Paczynski & Wiita 1980). If we fixed $l_*$, the condition of $d\psi / dr = d^2\psi / dr^2 = 0$ gave a critical radius

⇒ $r = 3\, r_g$ (= $r_{ms}$: Marginally stable orbit), $l_* = (3/2)3/2 c\, r_g$ (~1.83712 $c\, r_g$)

$$\psi_{eff}^{PN0}(r) = -\frac{GM}{r - r_g} + \frac{l_*^2}{2r^2}$$

See, e.g., Kato, Fukue & Mineshige 1998

Location of $r_{ms}$ is exactly same as general relativistic case.

# Disk model & method

- Basic equations : height integrated approximation (1D)

1. **Mass conservation**
2. **Momentum equation (-r)**
3. **Angular momentum**
4. **Hydrostatic balance**
5. **Equation of state**
6. **Energy balance**

$$1 : \dot{M} = -2\pi r \Sigma v_r$$

$$2 : v_r \frac{dv_r}{dr} + \frac{1}{\Sigma}\frac{dW}{dr} = \frac{l^2 - l_K^2}{r^3} - \frac{W}{\Sigma}\frac{d \ln \Omega_K}{dr}$$

$$3 : \dot{M}(l - l_{in}) = -2\pi r^2 T_{r\varphi}$$

$$4 : H = (p_{tot}/\rho\Omega)^{1/2}$$

$$5 : p_{tot} = p_{gas} + p_{rad}$$

$$6 : Q_{vis}^{+} = Q_{rad}^{-} + Q_{adv}$$

- **Viscosity : $\alpha$ - viscosity**

$$T_{r\varphi} = -\alpha\Pi = -\alpha p_{tot} H$$

**Method**

Semi-implicit Runge-Kutta method

$r_{out} = 10^4 r_g \rightarrow r_{in} \sim r_g$ through the transonic point

Parameter: $\alpha$=0.01, 0.1, $M$~10$M_{sun}$

$\Sigma$: surface density
$V_r$: radial velocity
$W$: height integrated pressure ($W=pH$)
$l_k$, $\Omega_k$: Kepler angular momentum, angular velocity
$l_{in}$ : Angular momentum at transonic point
$H$: scale height
$p$: pressure
$\rho$ : density

# Effective potential distribution

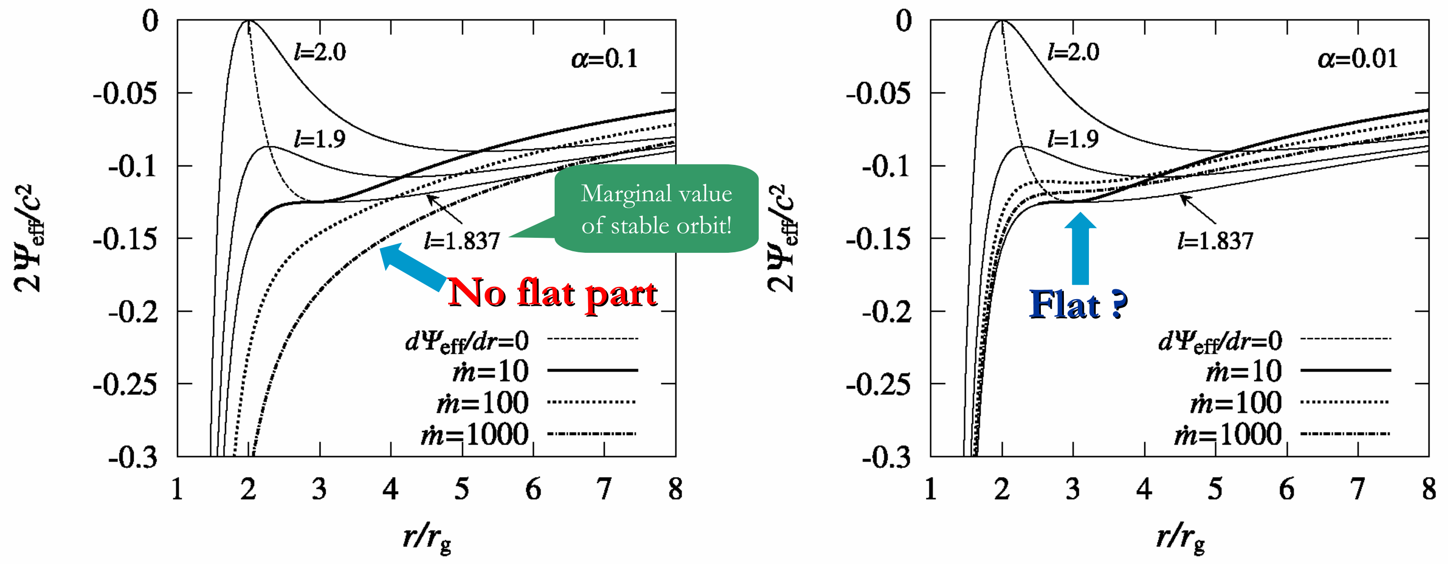



**Fig. 1.** Profiles of the effective potentials for $\alpha = 0.1$ (left panel) and $\alpha = 0.01$ (right panel), respectively. The thick solid, dotted, dashed and dotted lines represent the numerically calculated potential for accretion rates of $\dot{m}\ (\equiv \dot{M}c^2/L_E) = 10$, 100, and 1000 from the top to bottom. The thin solid lines show the same results but for a fixed specific angular momentum of $\ell\ (\equiv r v_\varphi) = 2.0$, 1.9, and 1.837 from the top to bottom. The thin dotted line represents the $\psi_{\rm eff} = 0$ curve.

- For larger $\alpha$ (=0.1) : no flat part for high accretion rate

→ no stable orbit!

- For smaller $\alpha$ (=0.01) : flat part appear

→ stable orbit ?

# Effect of Pressure Gradient

- Effect of pressure gradient is important for $\alpha$ =0.01, since the location of maximum pressure is about 3 $r_g$. We modify the effective potential as

$$\psi_{eff}^{PN1}(r) = -\frac{GM}{r-r_g} + \frac{l_*^2}{2r^2} - \int_{r_{in}}^{r_{out}} \frac{1}{\Sigma}\left(\frac{dW}{dr}\right) dr.$$

Here, the third term describes the change in the internal energy by work done by the pressure gradient force.

Direction of force

- dP/dr >0 : inward ( < 3 $r_g$)
- dP/dr <0 : outward (> 3 $r_g$)

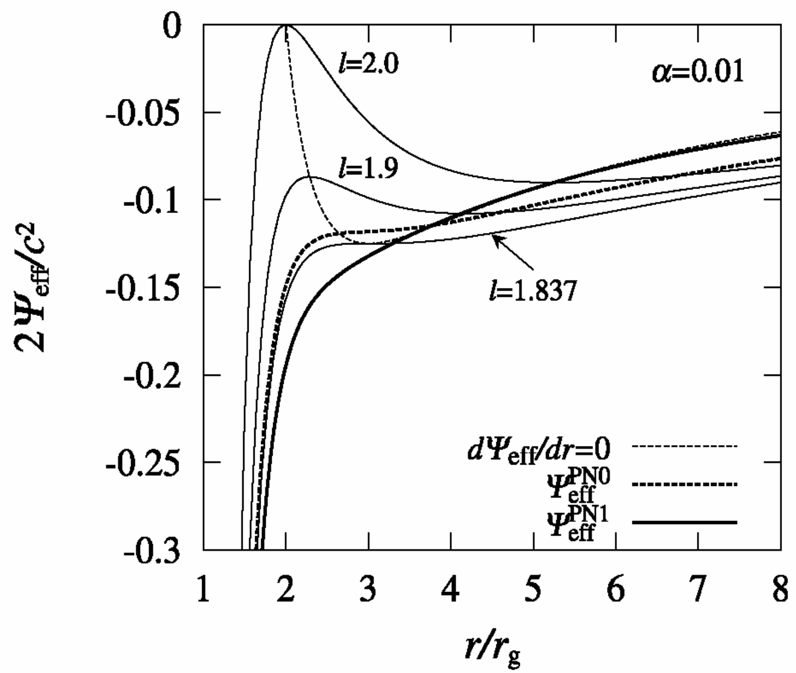



**Fig. 2.** Profiles of the effective potentials for $\alpha = 0.01$. The thick solid line represent the numerically calculated potential, equation (13), for accretion rates of $\dot{m}$ $(\equiv \dot{M}c^2/L_E) = 1000$. The thick dotted line show the calculated potential, but the line are derived from equation (12).

- For small $\alpha$, the flow is driven by pressure! →no marginally stable orbit!
- For large $\alpha$, the flow driven by viscosity.

All of these results are consistent with the results of Matsumoto et al. 1984, Narayan et al. 1997

# ADAF case

- Optically thin ADAF also does not have a marginally stable orbit, since ADAF does not have enough angular momentum to make a stable circular orbit.

⇒ gravitational red-shift : photon escape fraction ~30% at $2r_g$.

⇒ How about Iron K$\alpha$ line profile?

(see also Kawanaka's poster!)

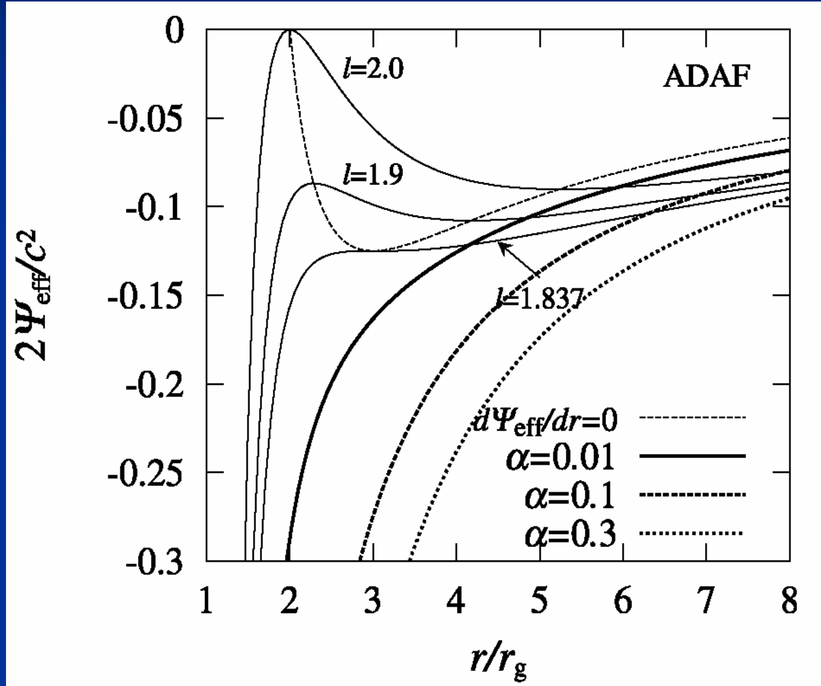
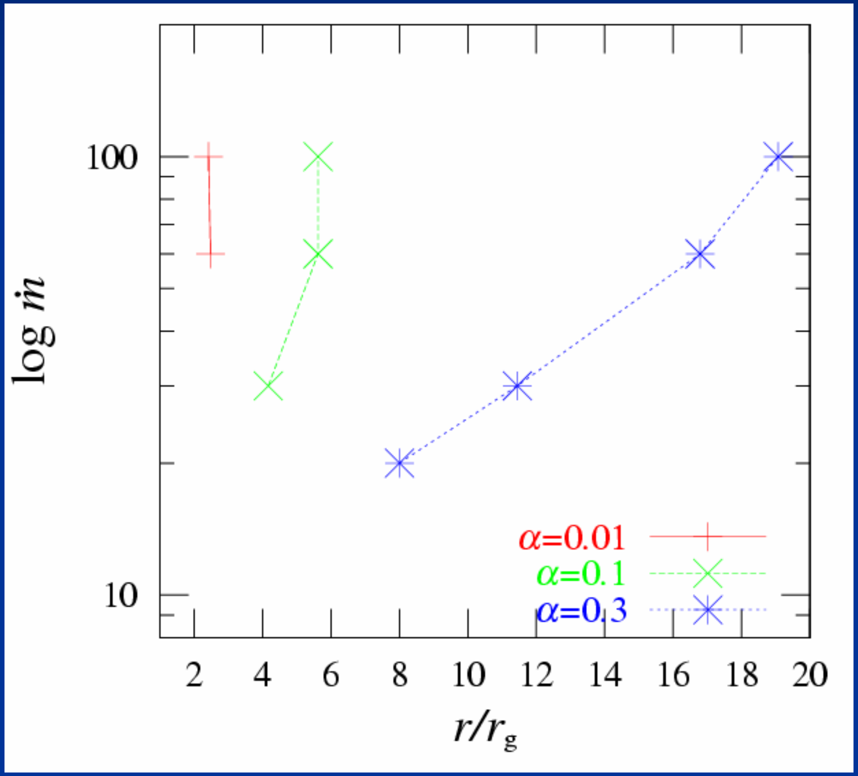



**Fig. 3.** Same as figure 1, but for an optically thin ADAF. The solid lines represent the effective potentials for $\alpha = 0.01, 0.1, 0.3$ from the top to bottom. We set the accretion rate to be $\dot{m} = 10^{-3}$. There is slight accretion rate dependence.

# Where is Radiation Edge?




- We plot $\tau_{eff}$ =1 radius.

⇒ transition radius form optically thick to optically thin.

(hereafter, Radiation edge, $R_{rad}$)

- Low Mdot : $R_{rad} \sim r_{ms}$
- High Mdot : $R_{rad} > r_{ms}$

- That is, "the inner edge" can be changed by the definition (Krolik & Hawley 2002).

From observation: peak temperature of thermal spectrum in black hole candidates or QPO frequency → We can see "radiation edge" from observation, however, the edge location is changed by the accretion rate and the magnitude of the viscosity!

# Discussion

- The interpretation of the observed QPO frequencies should be re-considered, since the assumption of a Keplerian rotation velocity can grossly over- or underestimate the disk rotation velocity, depending on the magnitude of the viscosity.

- Observational interpretation as to the black hole rotation: If the inner disk radius, $R_{in}$, estimated by spectral fitting less than $3r_g$ ⇒black hole rotation? (Zhang et al. 1997; Makishima et al. 2000). This argument does not hold for luminous systems shining at $L \sim L_E$, since they always show a smaller inner-edge radius.

- Recently, multi-dimensional Magneto-Hydrodynamic (MHD) simulations have succeeded to confirm the presence of a marginally stable orbit at around $\sim 3r_g$ (Armitage et al. 2001; Hawley, Bulbus 2002). On the other hand, Krolik and Hawley (2002) proposed that the "inner edge" of an accretion disk depends on the definition, i.e., turbulence edge, stress edge, reflection edge, and radiation edge are defined by each physical process. These edges does not need to coincide with $r_{ms}$.

# Conclusions

- We discuss the dynamical properties of transonic flow in super-critical regimes (Mdot > $10L_E/c^2$) and demonstrate that a marginally stable orbit does not exist for high accretion rates.

1. For small-viscosity parameters, the effect of a pressure gradient is important for discussing the marginally stable orbit.→The inner disk radius obtained by the spectral fitting does not necessarily coincide with the radius of a marginally stable orbit.
2. To determine the radiation edge, we need to solve full radiation-transfer equations in more than two dimensions, which is left as future work.