相対論的球対称風における
相対論的輻射輸送の解き方

Relativistic Radiative Transfer
Relativistic Formal Solution
Spherical Flow

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目次

－１　何がいいたいか

・天体風（新星・超新星）は球対称には見えない
・（相対論的）輻射輸送は面白い宝の山

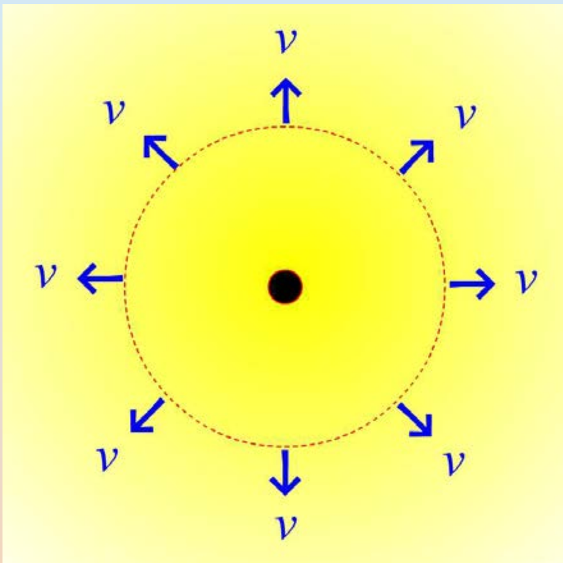
Bath and Shaviv 1976

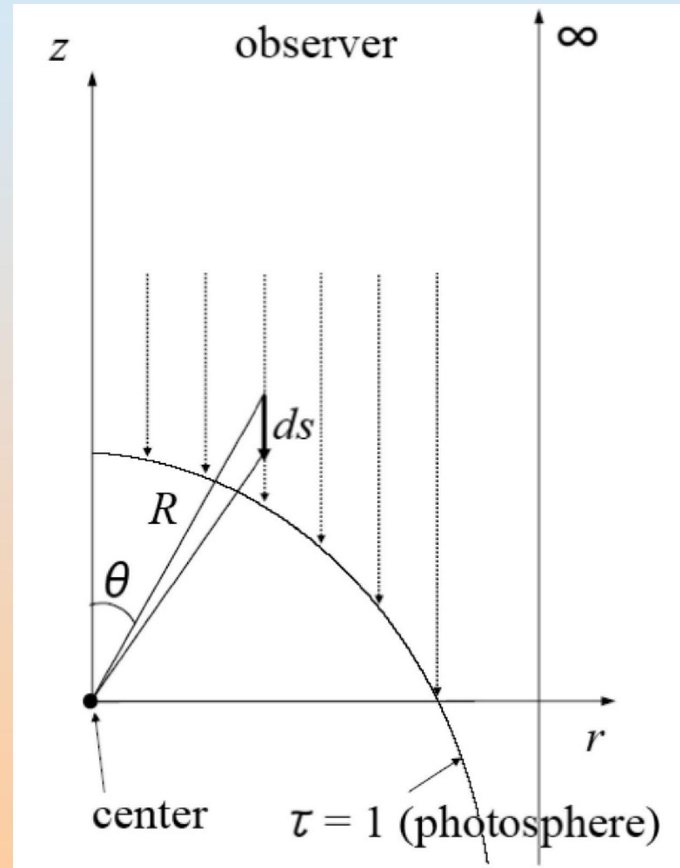
A crude estimate of the position of the photosphere (assumed to be at $\tau = \frac{2}{3}$) is thus

$$r_s = \frac{3\kappa\dot{m}}{8\pi v_s} \qquad (2)$$

where r_s and v_s are the radius and the velocity of matter through the photosphere. Note that this is a purely optical photosphere. Matter is continually passing outward through it, in such a way as to satisfy the condition of steady state continuity (equation (1)). Of course the photosphere will *not* be a sharply-defined surface, unlike the case of static stellar models for which the equivalent condition to (2) is $P\kappa = \frac{2}{3}g$. Limb-darkening effects will be large, and non-steady conditions may be important. Furthermore, as Friedjung (1966a) has discussed in some detail, if electron scattering dominates then a correction must be included in relation (2) to account for the fact that scattering increases the time spent by a photon in diffusing through the gas and hence enhances the probability of an absorption occurring. This may be crudely treated (e.g. Tucker 1967; Felton & Rees 1972) by taking a geometric mean of the absorption, κ_{ab} and scattering, κ_{sc}, opacity (i.e. $\kappa \sim (3\kappa_{ab}\kappa_{sc})^{1/2}$). This clearly reduces r_s. On the other hand the inclusion of curvature effects in the derivation of (2) would increase r_s by a factor 3 due to the introduction of a term $(r_s/r)^2$ in the integral determining τ (see, for example, Lucy 1976). For the moment we ignore both effects but return to discuss them further in Section 5.

見かけの光球（擬光球）

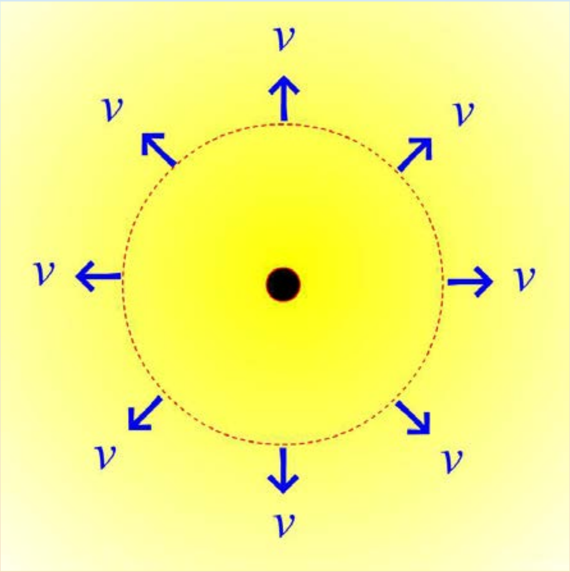


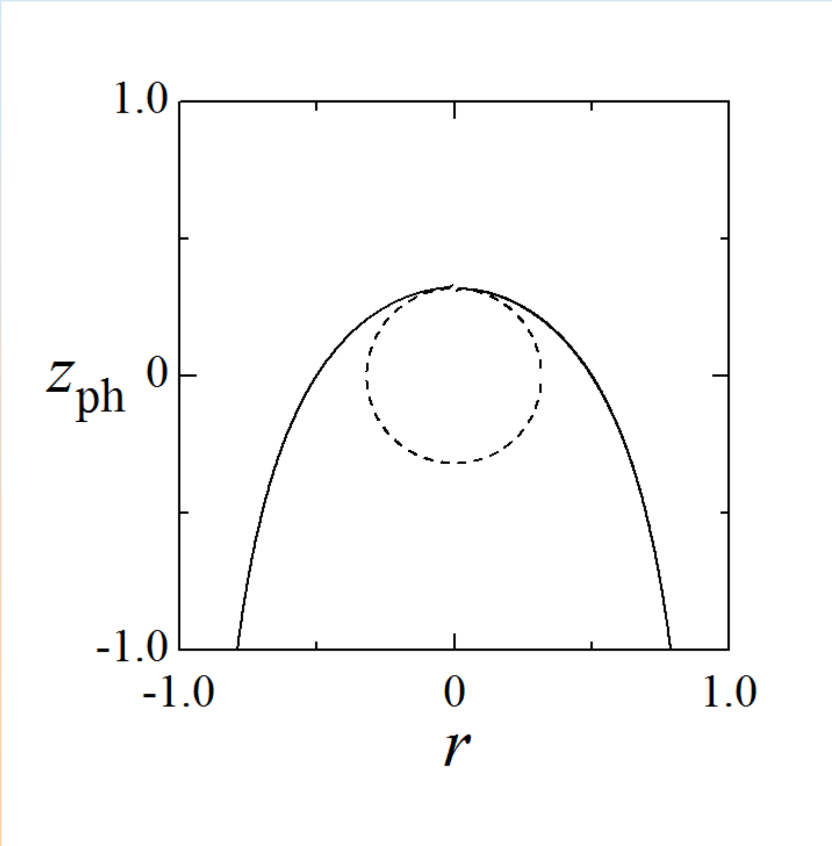


$$R_{\mathrm{photo}} \equiv\equiv -\int_{\infty}^{R} (\kappa+\sigma)\rho\, dR = 1$$

$$z_{\mathrm{ph}} \equiv\equiv -\int_{\infty}^{z} (\kappa+\sigma)\rho\, dz = 1$$

見かけの光球（擬光球）

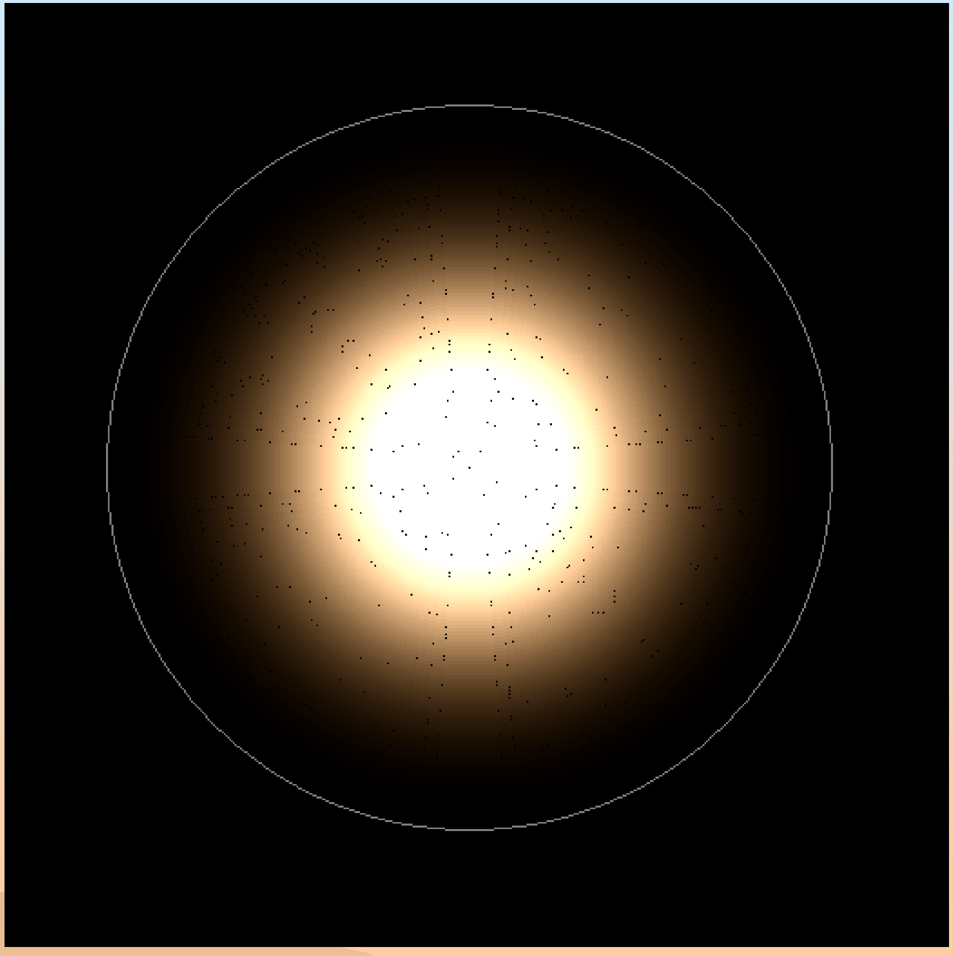




$$R_{\rm ph} = \frac{\kappa_{\rm es}\dot{M}}{4\pi v} = \frac{\dot{m}}{2\beta} r_{\rm g}$$

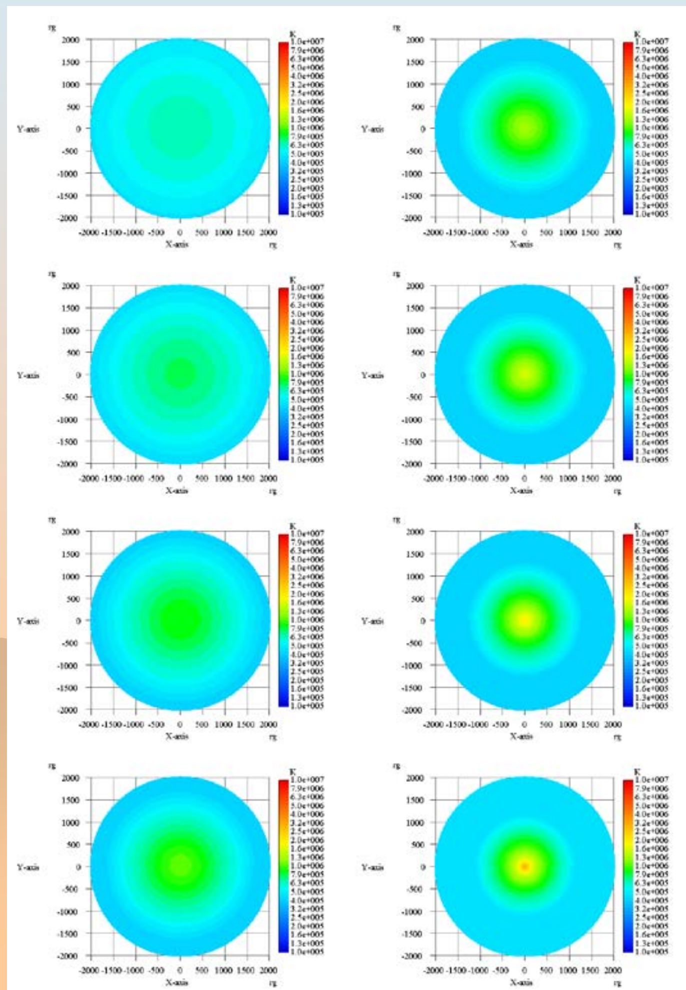
$$\frac{z_{\rm ph}}{r} = \tan\left(\frac{\pi}{2} - \frac{2\beta}{\dot{m}}\frac{r}{r_{\rm g}}\right)$$

周縁減光効果

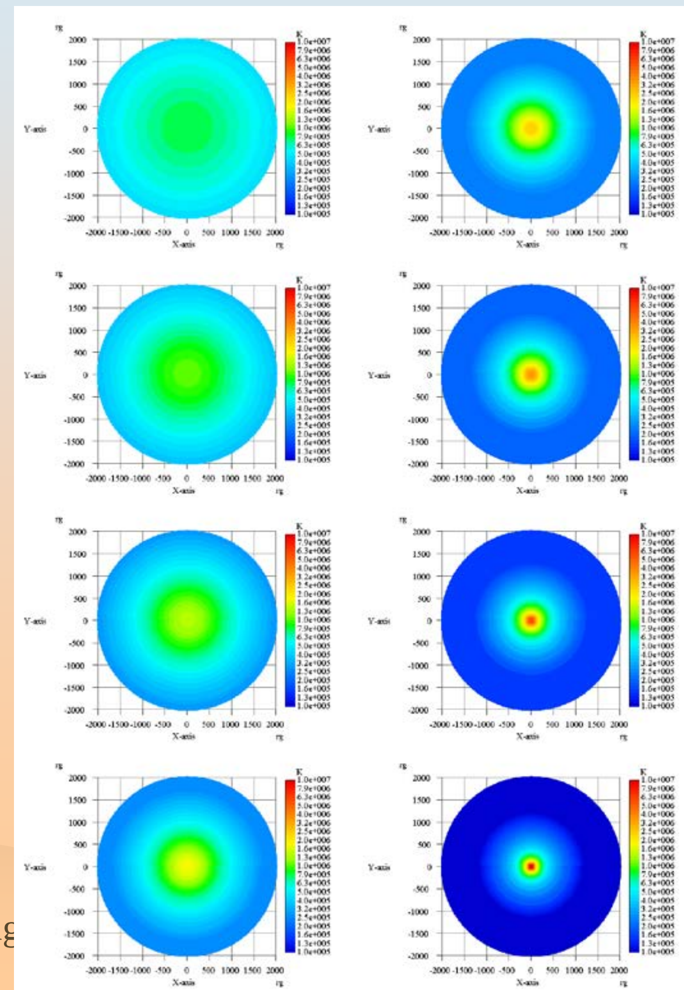


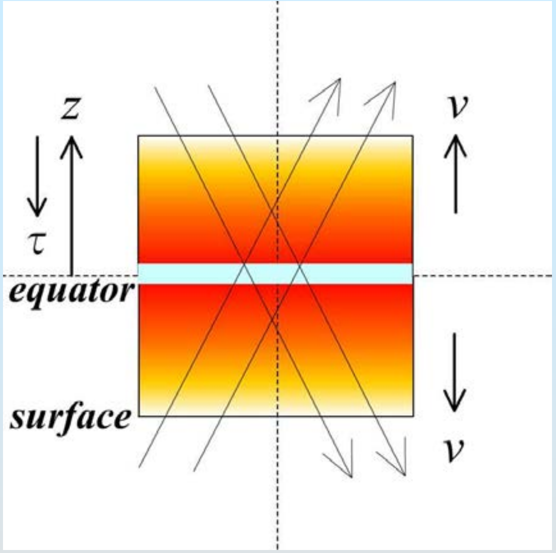
相対論的周縁減光効果

共動系:0.2~0.9c

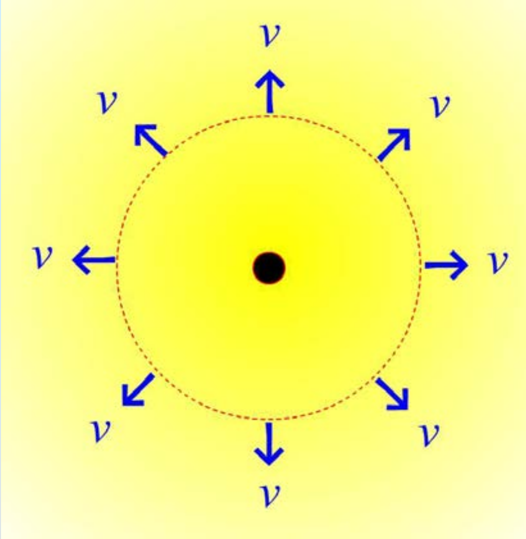


静止系:0.2~0.9c





０　やったこと



- 相対論的平行平板流の相対論的形式解の導出
  - 速度場を与えて相対論的輻射輸送を解く

    Fukue 2014
  - 速度場と輻射場を同時に解く

    Fukue 2015
- 相対論的球対称流の相対論的形式解の導出
  - 速度場を与えて相対論的輻射輸送を解く

    Fukue 2017?

１　輻射圧で駆動される相対論的球対称流

1 Relativistic Radiation Hydrodyanmics

相対論的輻射流体力学の定式化

Thomas, L.H. 1930, Quart. J. Math 1, 239
Hazlehurst, J., Sargent, W.L.W. 1959, ApJ 130, 276
Lindquist, R.W. 1966, Annals Phys. 37, 487
Castor, J.I. 1972, ApJ 178, 779
Anderson, J.L., Spiegel, E.A. 1972, ApJ 171, 127
Hsieh, S.-H., Spiegel, E.A. 1976, ApJ 207, 244
Thorne, K.S. 1981, MNRAS 194, 439
Udey, N., Israel, W. 1982, MNRAS 199, 1137
Mihalas, D., Klein, R.I. 1982, J.Comp.Phys. 46, 97
Mihalas, D., Mihalas, B.W. 1984, Foundations of Radiation Hydrodynamics (Oxford University Press)
Park, M.-G. 1993, A&A 274, 642
Mihalas, D., Auer, L.H. 2001, JQSRT 71, 61

一般相対論的輻射流体力学の方程式系が成分で書き下されたのはごく最近

Park, M.-G. 2006, MNRAS 367, 1739
Takahashi, R. 2007, MNRAS 382, 1041

Closure 問題

Usual closure relation for radiation

Eddington approximation

$$P_{\rm co}^{ik} = \frac{\delta^{ik}}{3} E_{\rm co}$$

Diffusion approximation

$$F_{\rm co}^{i} = -\frac{c}{\kappa_R \rho}\frac{\partial P_{\rm co}^{ik}}{\partial x^k} = -\frac{c}{3\kappa_R \rho}\frac{\partial E_{\rm co}}{\partial x^i}$$

Flux - limited diffusion

$$F_{\rm co}^{i} = -\lambda \frac{c}{\kappa_R \rho}\frac{\partial E_{\rm co}}{\partial x^i}$$

- エディントン因子

Fukue 2005

- 拡散近似

Castor 1972
Ruggles, Bath 1979
Flammang 1982
Tullola+ 1986
Paczynski 1990
Nobili+ 1993, 1994

- シミュレーション

Eggum+ 1985, 1988
Kley 1989
Okuda+ 1997
Kley, Lin 1999
Okuda 2002
Okuda+ 2005
Ohsuga+ 2005
Ohsuga 2006

Closure 問題

Usual closure relation for radiation

In the comoving frame

Eddington approximation

$$P_{\mathrm{co}}^{ik} = \frac{\delta^{ik}}{3} E_{\mathrm{co}}$$

Diffusion approximation

$$F_{\mathrm{co}}^{i} = -\frac{c}{\kappa_R \rho}\frac{\partial P_{\mathrm{co}}^{ik}}{\partial x^k} = -\frac{c}{3\kappa_R \rho}$$

Flux - limited diffusion

$$F_{\mathrm{co}}^{i} = -\lambda \frac{c}{\kappa_R \rho}\frac{\partial E_{\mathrm{co}}}{\partial x^i}$$

- エディントン因子

$v = c/\sqrt{3}$で病的な特異性が出現

相対論的な領域では、
共動系でも等方的でなくなる

Nobili+ 1993, 1994

- シミュレーション

非因果的
静止大気以外での適用は危険

Ohsuga 2006

2. RRHD Closure Relation 2

What is a closure relation in subrelativistic to relativistic regimes

* Velocity - dependent

  variable Eddington factor

  $P_{co} = f(\beta) E_{co}$

  • plane - parallel

  $$f(\beta) = \frac{1+2\beta}{3}; \quad \beta = \frac{v}{c}$$

  Fukue 2006

  • spherical

  $$f(\tau, \beta) = \frac{1+\tau/[\gamma(1+\beta)]}{1+3\tau/[\gamma(1+\beta)]}$$

  Akizuki, Fukue 2007

* Numerical simulation

  $f(\tau, \beta)$

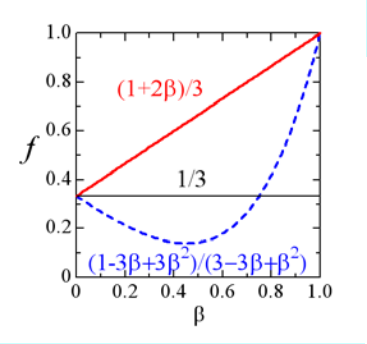
  Koizumi, Umemura 2007

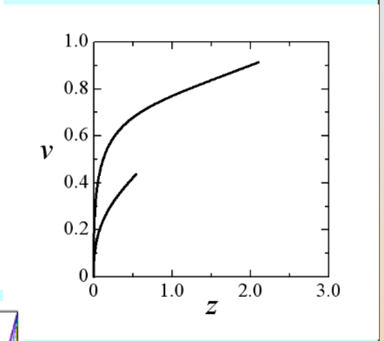
* Velocity - gradient - dependent

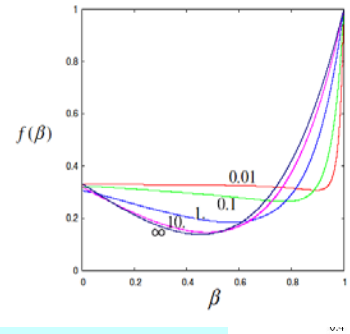
  variable Eddington factor

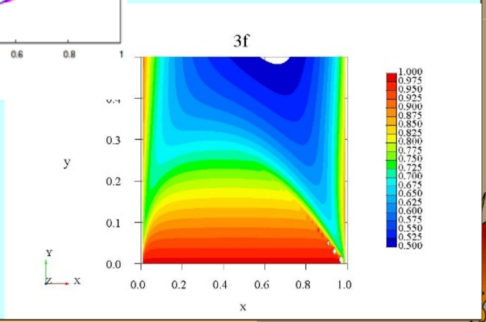
  $f(\tau, \beta, d\beta / d\tau)$

  Fukue 2007

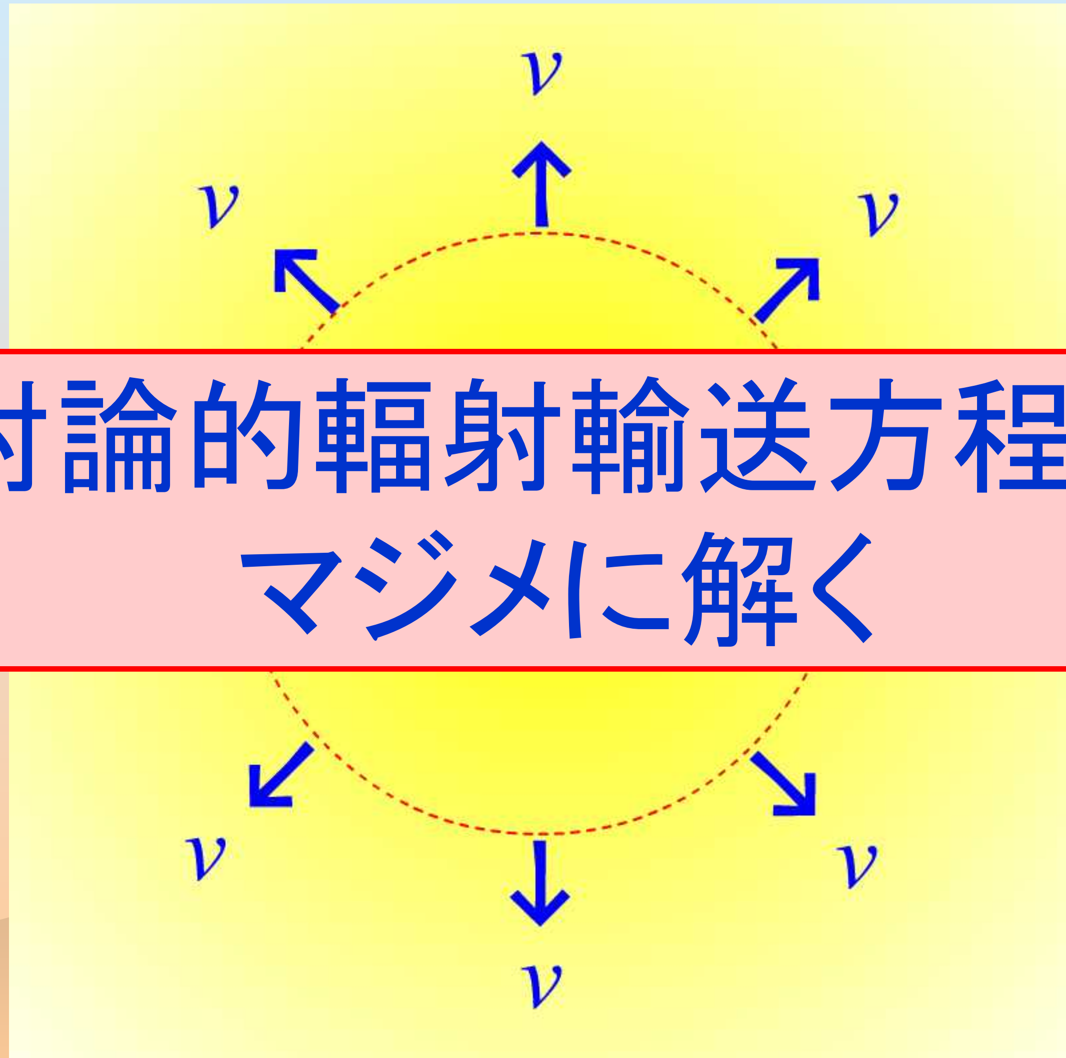








相対論的球対称輻射流



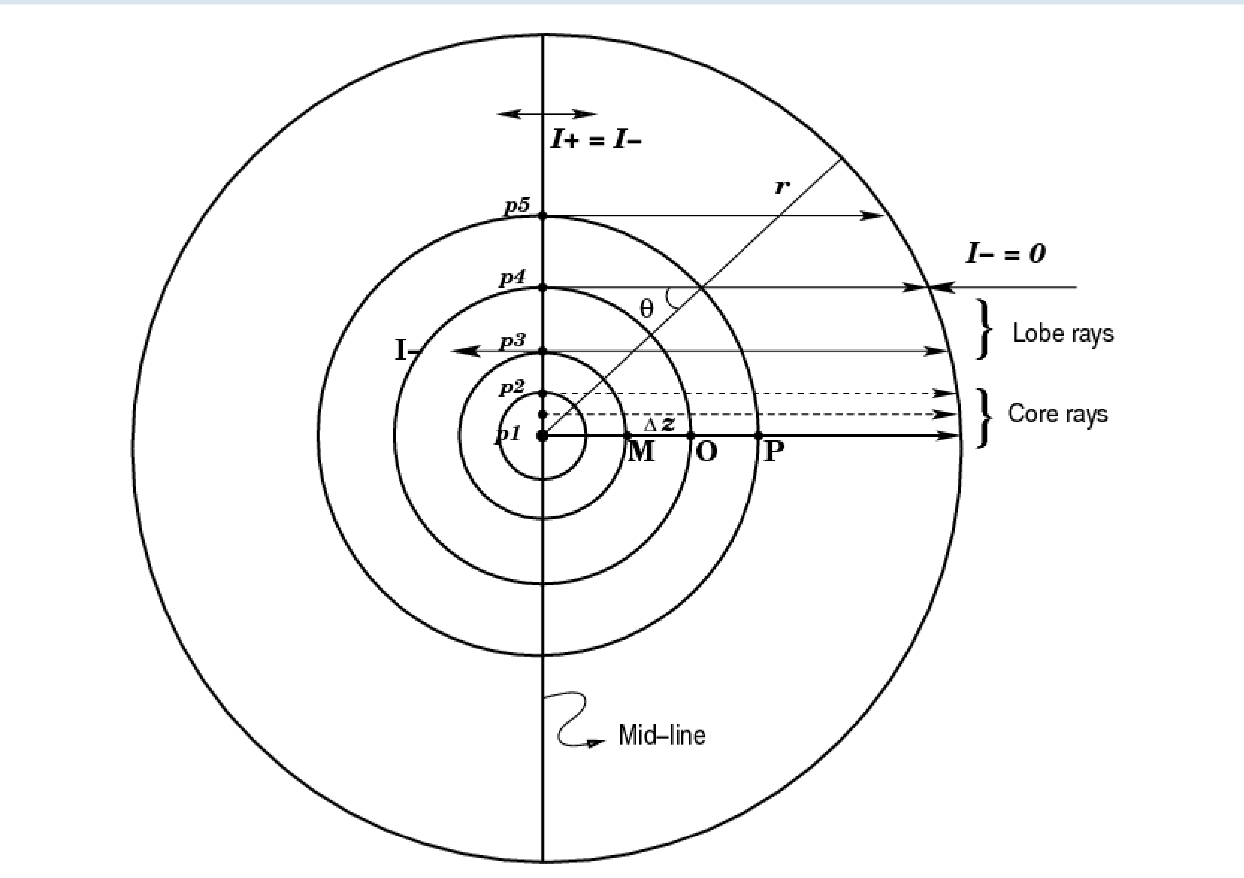
**相対論的輻射輸送方程式を
マジメに解く**

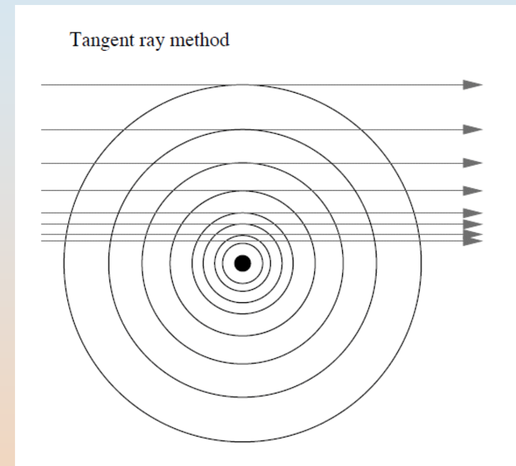
２　相対論的球対称流における相対論的輻射輸送方程式の形式解

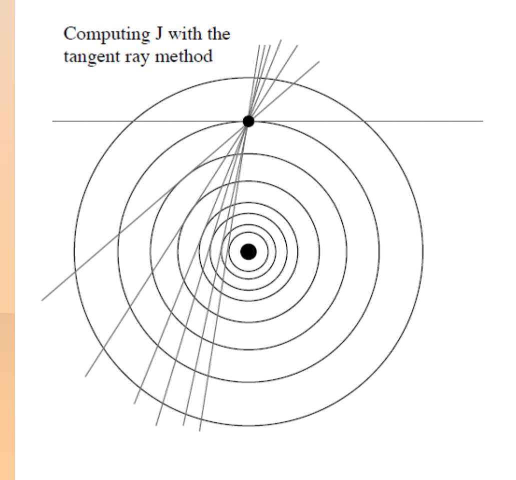
Relativistic Formal Solutions of Relativistic Radiative Transfer Equation in Relativistic Spherical Flows

Impact Parameter Method
Tangent Ray Method

- Hummer and Rybicki 1971

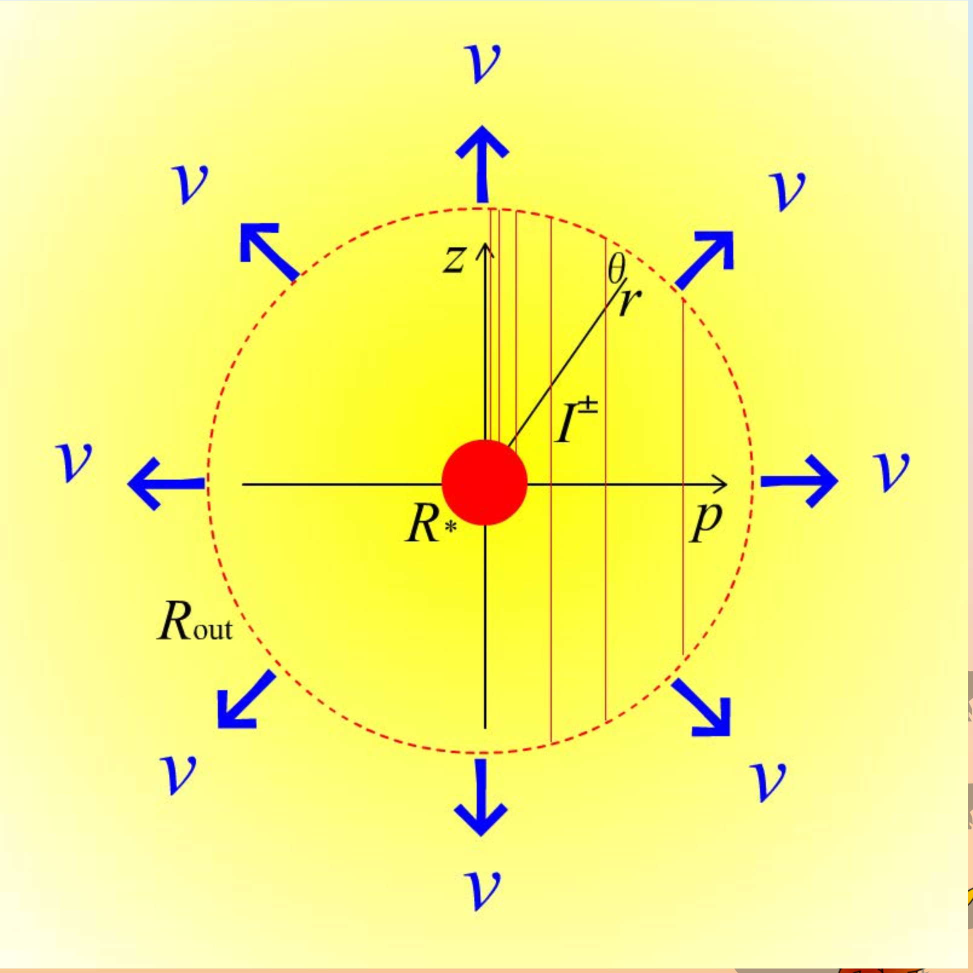




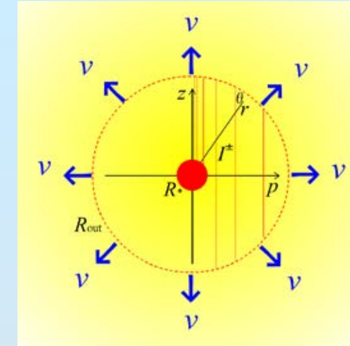


相対論的球対称流

- 定常一次元球対称流
- 速度は与える
  - 一定
  - 自由落下
- 灰色近似
- $v(r)$, $\rho(r)$, $S(r)$
- $I^+(p,z)$, $I^-(p,z)$
- $r^2=p^2+z^2$



相対論的輻射輸送方程式



- 定常、一次元、球対称流
- 等方散乱

混合系

$$\pm\frac{\partial I^{\pm}(p,z)}{\partial z}=\frac{\rho_0}{\gamma^3(1-\boldsymbol{\beta}\cdot\boldsymbol{l})^3}\left[\frac{j_0}{4\pi}-(\kappa_0+\sigma_0)\,I_0+\sigma_0 J_0\right],(1)$$

- 変換

$$I_0=\gamma^4(1-\boldsymbol{\beta}\cdot\boldsymbol{l})^4 I, \quad (2)$$

$$\boldsymbol{\beta}\cdot\boldsymbol{l}=\pm\beta(r)\cos\theta=\pm\beta(r)\frac{z}{r} \quad (3)$$

$$J_0=\gamma^2\left(J-2\beta H+\beta^2 K\right), \quad (4)$$

$$H_0=\gamma^2\left[\left(1+\beta^2\right)H-\beta(J+K)\right], \quad (5)$$

$$K_0=\gamma^2\left(\beta^2 J-2\beta H+K\right), \quad (6)$$

$$f\equiv\frac{K_0}{J_0}. \quad (7)$$

相対論的輻射輸送方程式

- 源泉関数、光子破壊確率

$$\pm \frac{\partial I^{\pm}(p,z)}{\partial z} = -\frac{(\kappa_0+\sigma_0)\rho_0}{\gamma^3(1-\boldsymbol{\beta}\cdot\boldsymbol{l})^3}\left[\gamma^4(1-\boldsymbol{\beta}\cdot\boldsymbol{l})^4 I^{\pm} - S_0\right], \tag{8}$$

where

$$S_0(r) = \frac{j_0}{4\pi}\frac{1}{\kappa_0+\sigma_0} + \frac{\sigma_0}{\kappa_0+\sigma_0}J_0 \tag{9}$$

$$S_0 = \varepsilon_0 B_0 + (1-\varepsilon_0)J_0,$$

$$\varepsilon_0 \equiv \kappa_0/(\kappa_0+\sigma_0)$$

相対論的形式解

- 非相対論と同様に形式的に積分

$$X \equiv \exp\left[+\int^{z}(\kappa_0+\sigma_0)\rho_0\gamma d\zeta - \int^{z}(\kappa_0+\sigma_0)\rho_0\gamma\boldsymbol{\beta}\cdot\boldsymbol{l}d\zeta\right], \quad (12)$$

we rearranged equation (10) as

$$\pm\frac{\partial}{\partial z}\left(XI^{\pm}\right) = X\frac{(\kappa_0+\sigma_0)\rho_0}{\gamma^3\left(1-\boldsymbol{\beta}\cdot\boldsymbol{l}\right)^3}S_0, \quad (13)$$

相対論的形式解

- 非相対論と同様に形式的に積分

$$e^{G(p,\zeta)-U(p,\zeta)} I^{\pm}\Big|^{z} = \int^{z} X \frac{(\kappa_0+\sigma_0)\rho_0}{\gamma^3 (1-\boldsymbol{\beta}\cdot\boldsymbol{l})^3} S_0 d\zeta, \quad (14)$$

where

$$G(p,z) \equiv \int^{z} (\kappa_0+\sigma_0)\rho_0 \gamma d\zeta, \quad (15)$$

$$U(p,z) \equiv \int^{z} (\kappa_0+\sigma_0)\rho_0 \gamma \boldsymbol{\beta}\cdot\boldsymbol{l} d\zeta.$$

$$G(p,z) \equiv \tau_* \int^{z} \frac{\kappa_0+\sigma_0}{\kappa_*} \frac{\rho_0}{\rho_*} \gamma d\tilde{\zeta},$$

$$U(p,z) \equiv \tau_* \int^{z} \frac{\kappa_0+\sigma_0}{\kappa_*} \frac{\rho_0}{\rho_*} \gamma \boldsymbol{\beta}\cdot\boldsymbol{l} d\tilde{\zeta},$$

where $\tilde{\zeta} = \zeta / R_*$ and

$\tau_* = \kappa_* \rho_* R_*,$

相対論的形式解

- 非相対論と同様に形式的に積分

$$I^+(p,z) = e^{G(p,z_*) - G(p,z) - U(p,z_*) + U(p,z)} I^*(p,z_*)$$
$$+ \int_0^z \frac{e^{G(p,\zeta) - G(p,z) - U(p,\zeta) + U(p,z)}}{\gamma^3 \left(1 - \beta \frac{\zeta}{r}\right)^3}$$
$$\times (\kappa_0 + \sigma_0)\rho_0 S_0 d\zeta, \tag{17}$$

where $z_* \equiv \sqrt{R_*^2 - p^2}$ if the luminous core exists. On the other hand, integrating from $z_{\rm out}$ $(= \sqrt{R_{\rm out}^2 - p^2})$ to z, we have the downward intensity $I^-(p,z)$ as

$$I^-(p,z) = -\int_{z_{\rm out}}^z \frac{e^{[G(p,z) - G(p,\zeta)] + [U(p,z) - U(p,\zeta)]}}{\gamma^3 \left(1 + \beta \frac{\zeta}{r}\right)^3}$$
$$\times (\kappa_0 + \sigma_0)\rho_0 S_0 d\zeta, \tag{18}$$

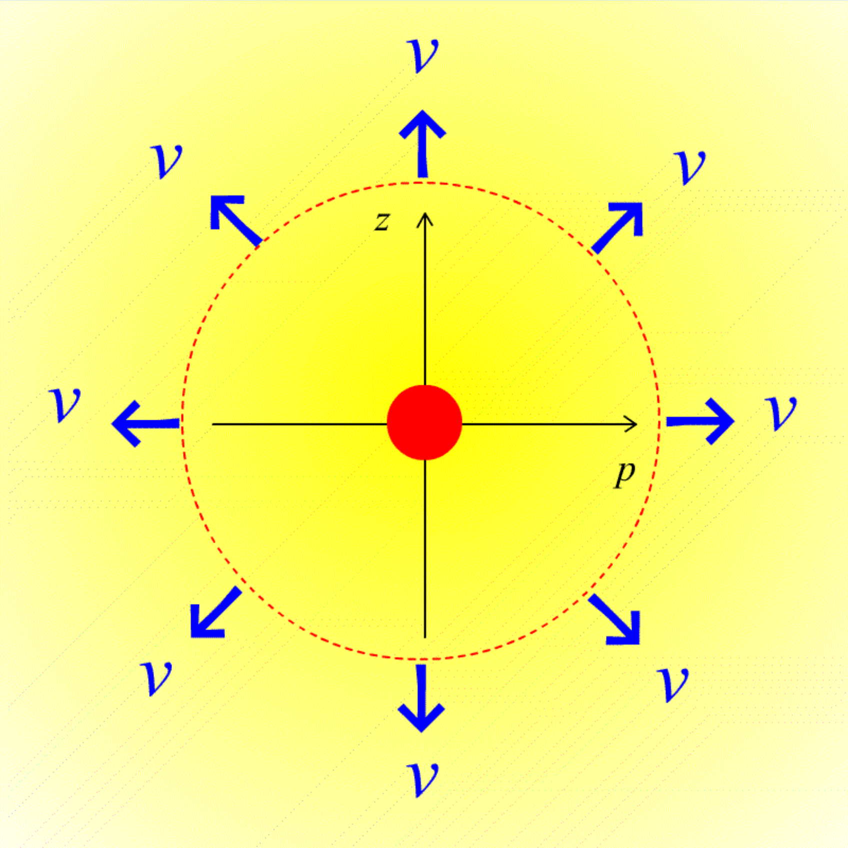
３　相対論的球対称における相対論的輻射輸送問題

Relativistic Radiative Transfer in Relativistic Spherial Flows

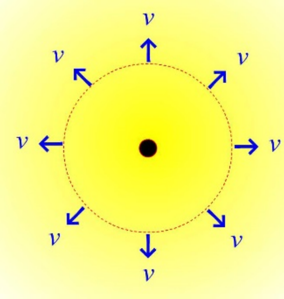
球対称風　1

- 球状光源：R^*、I^*
- 速度一定：$\beta = v/c$
- 散乱のみ：$S_0 = J_0$

- パラメータ
- β
- τ*



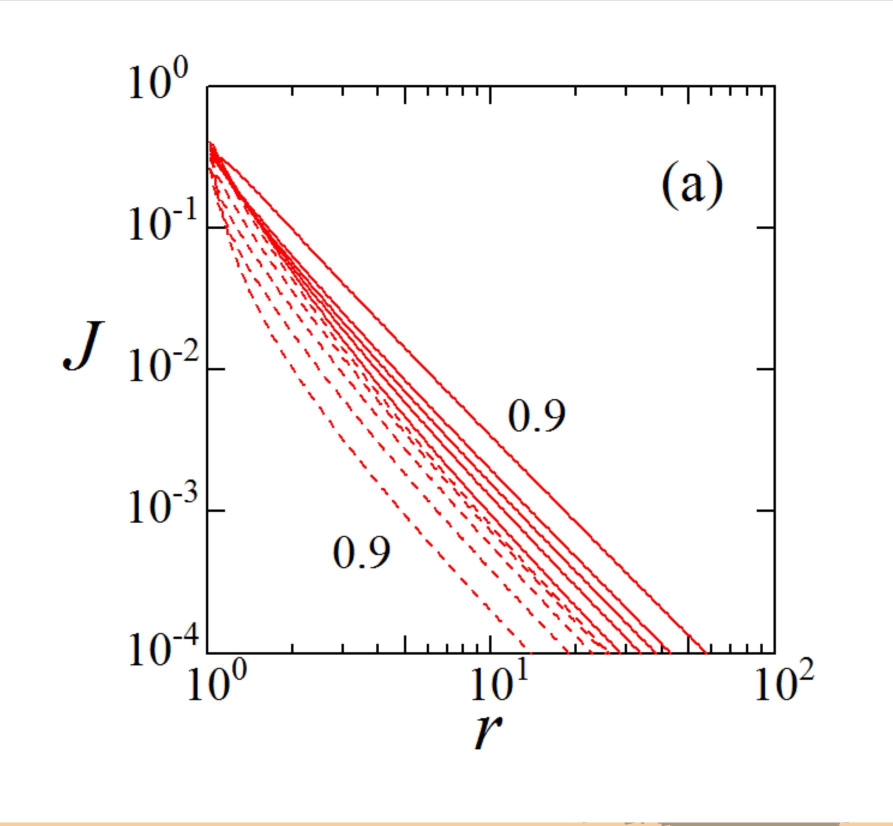
球対称風　1



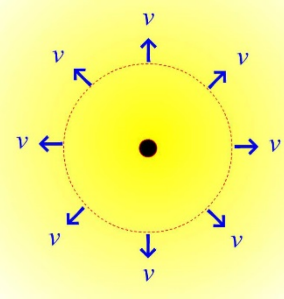
- 平均強度J（波線：共動系、実線：静止系）
- $\tau^*=3$（$R_{\mathrm{out}}=100R^*$）

- 低速

  $J \propto r^{-3} \sim r^{-2}$
- 高速



球対称風　1

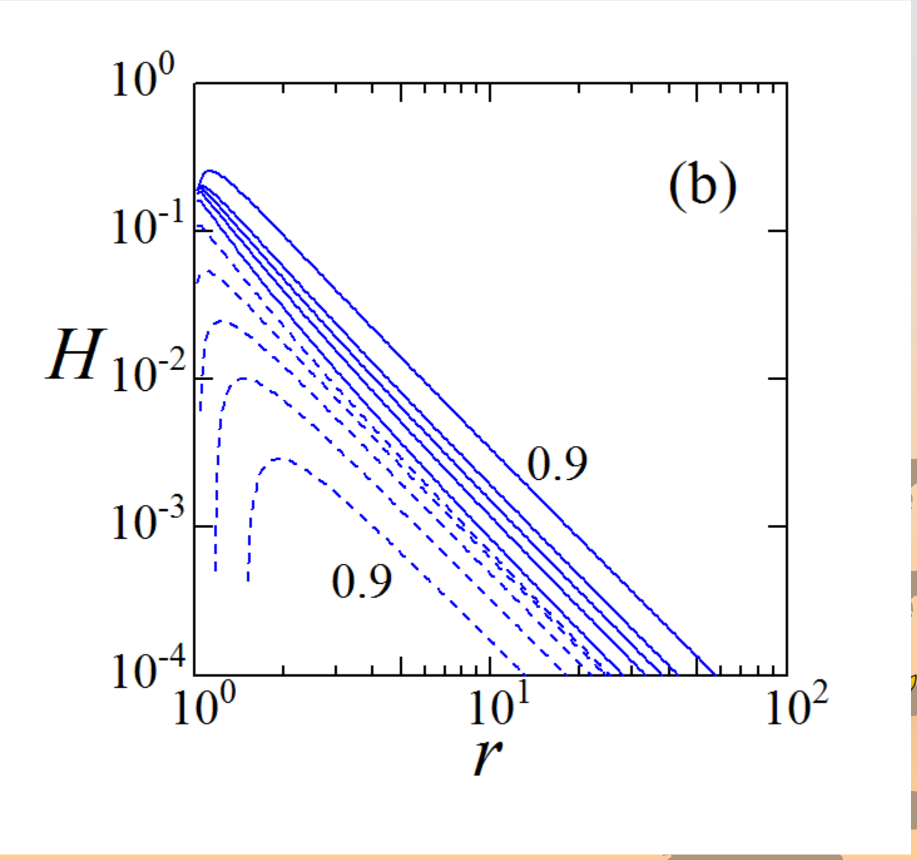


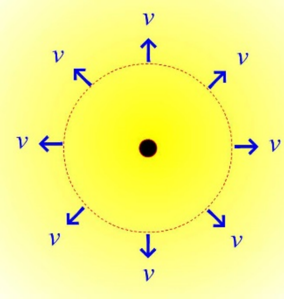
- 平均流束H（波線：共動系、実線：静止系）
- $\tau^*=3$（$R_{\rm out}=100R^*$）

- 低速

  $H \propto r^{-2}$

- 高速
  - ➢輻射抵抗の効果

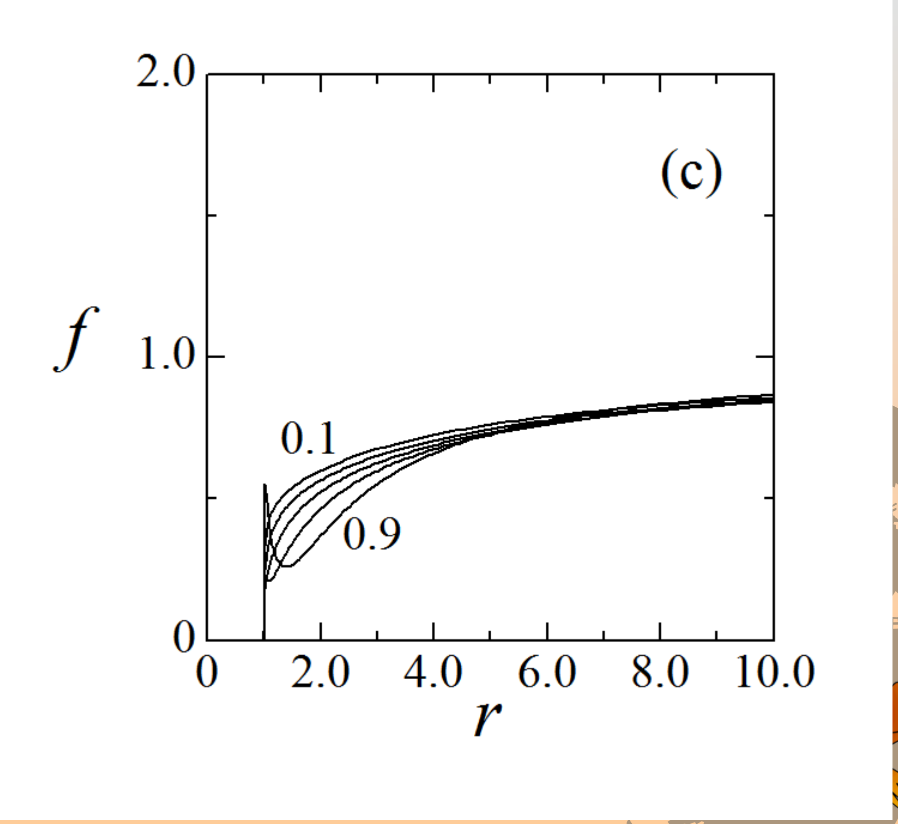




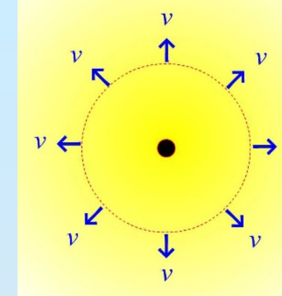
球対称風　1

- エディントン因子f
- $\tau^*=3$（$R_{out}=100R^*$）

- 低速
  $f=1/3\sim1$
- 高速
  $f=1\sim1/3\sim1$
  光行差

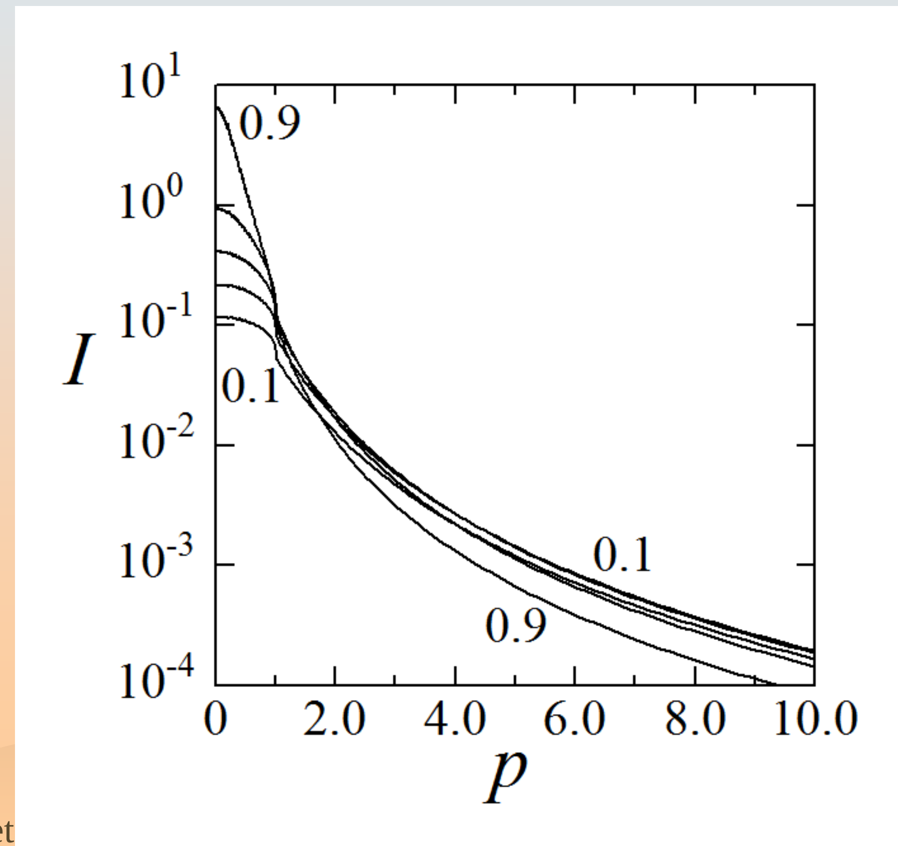


球対称風　１

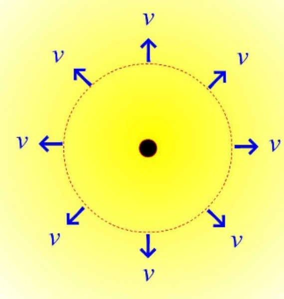


- 表面輝度$I^{+}(p, z_{out})$
- $\tau^{*}=3(R_{out}=100R^{*})$

- 低速
  コア~エンベロープ
- 高速
  ドップラー効果



球対称風　1



- 表面輝度$I^{+}(p, z_{\rm out})$
- τ^*=0.3, 10（$R_{\rm out}$=100R^*）

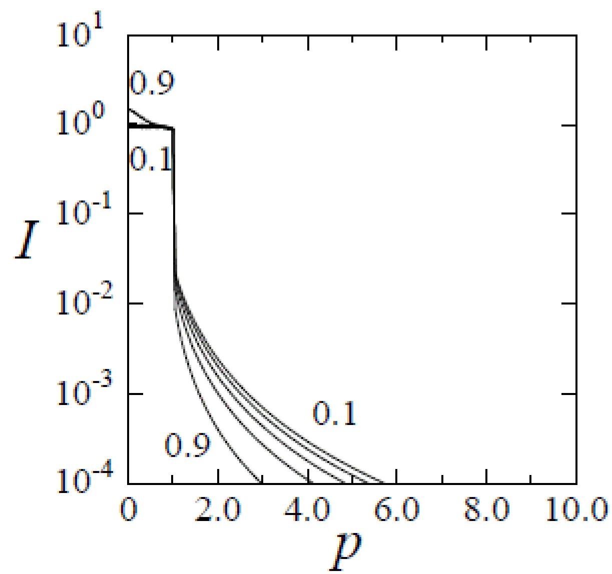


Fig. 4. Emergent intensity in the inertial frame at the outer radius $R_{\rm out}$ in the optically thin case of $\tau_* = 0.1$. The values of β are 0.1, 0.3, 0.5, 0.7, 0.9.

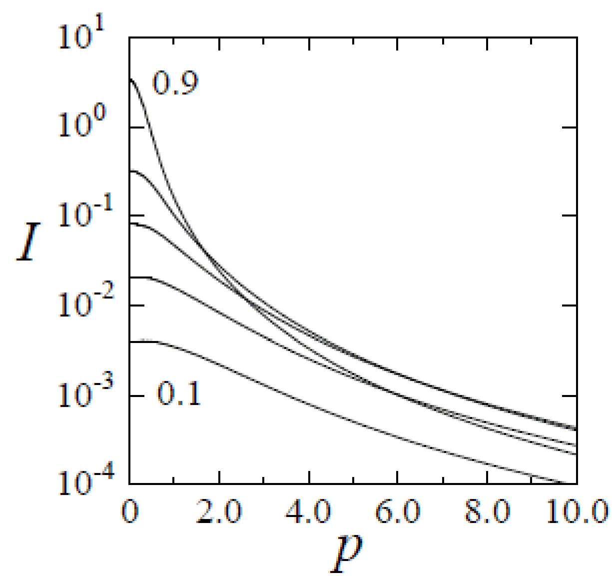
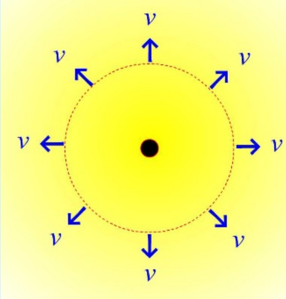
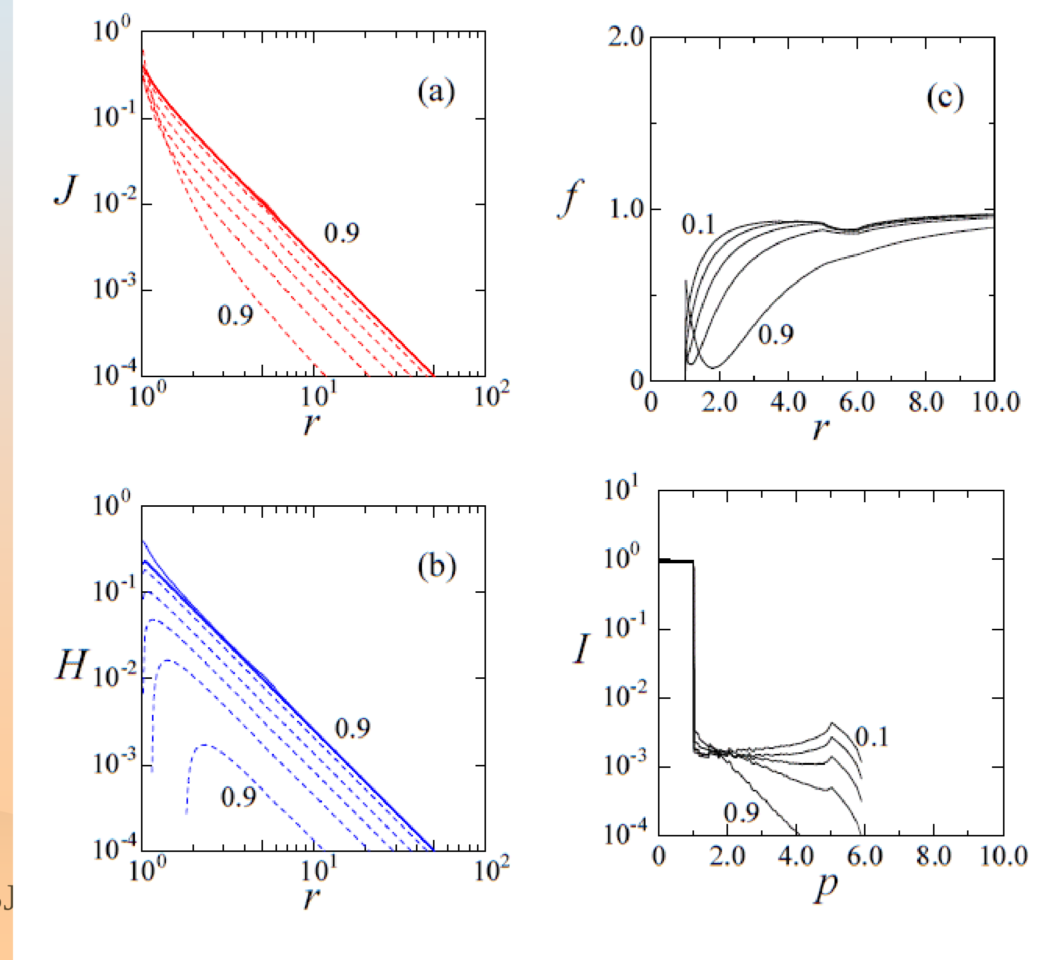


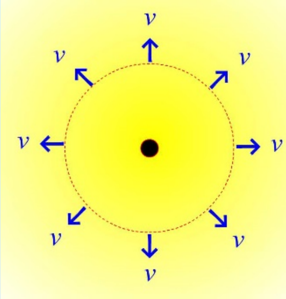
Fig. 5. Emergent intensity in the inertial frame at the outer radius $R_{\rm out}$ in the case of $\tau_* = 10$. The values of β are 0.1, 0.3, 0.5, 0.7, 0.9.

球殻風



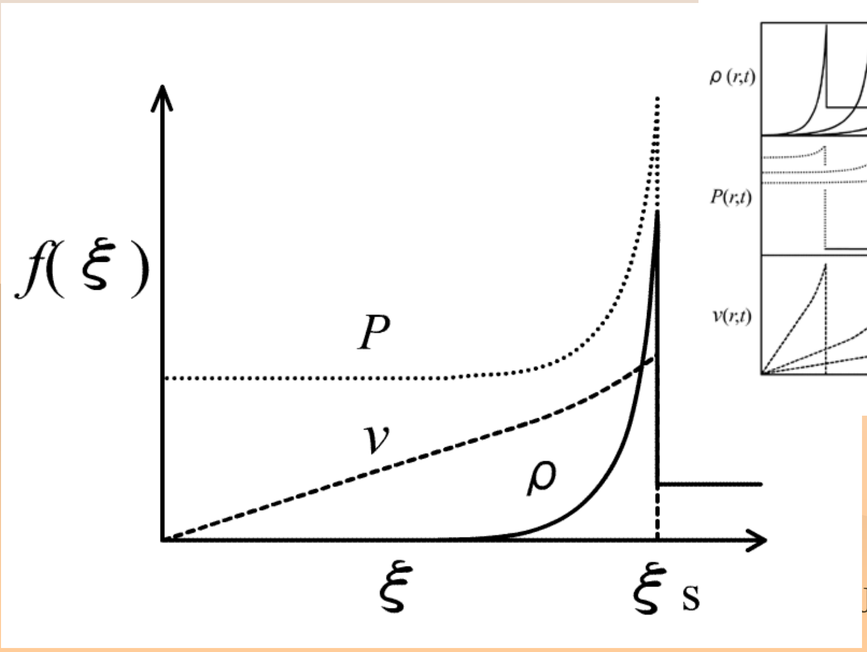
- 球殻（r=5-6）の場合
- τ^*=3（R_{out}=100R^*）
- J, H empty space

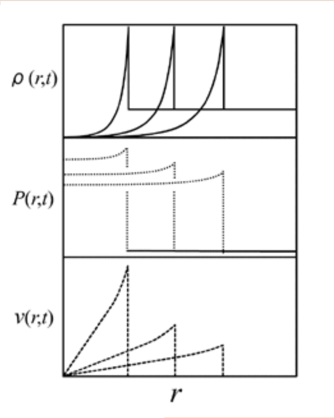


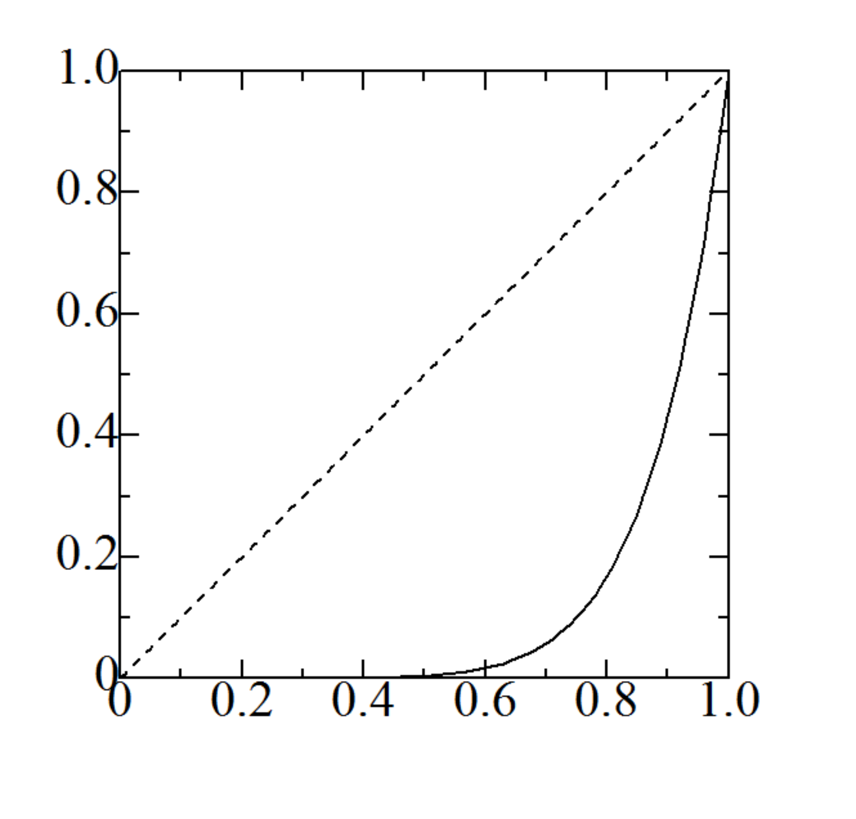


セドフ的流れ

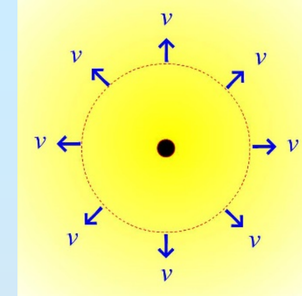
- セドフ解的な流れの場合
- $\tau^*=3$（$R_{\rm out}=100R^*$）



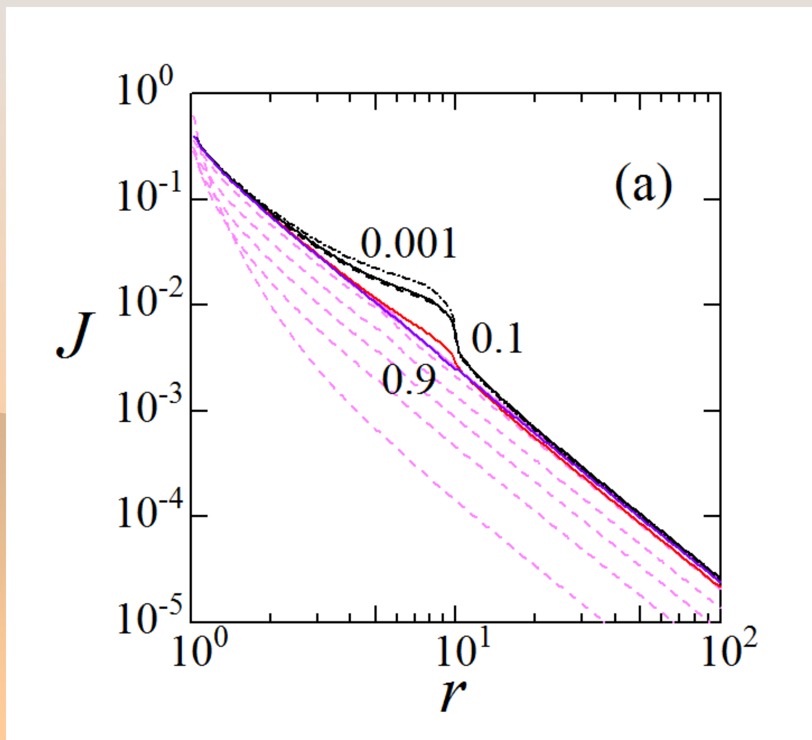


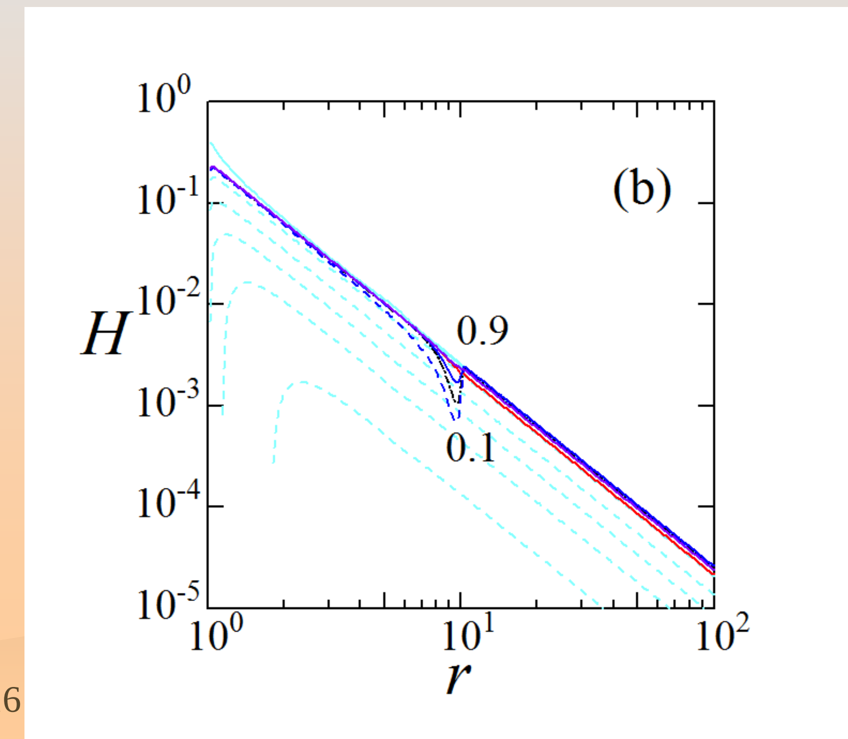


セドフ的流れ　１

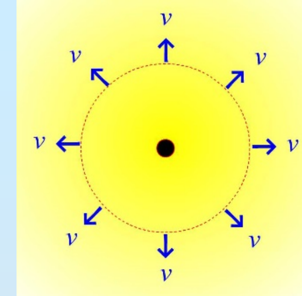


- セドフ的な流れの場合（R_{max}=10）
- τ^*=3（Rout=100R^*）

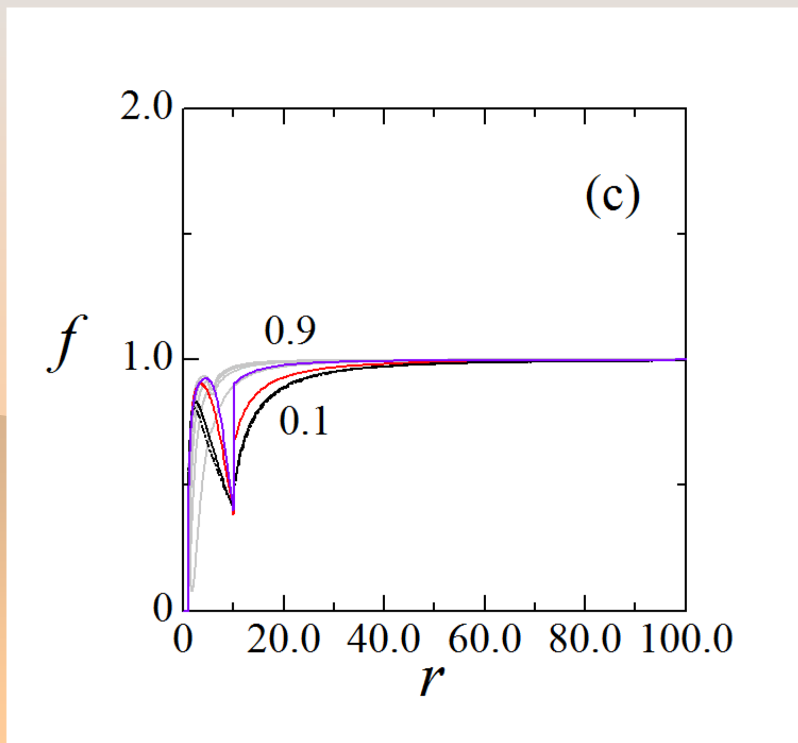


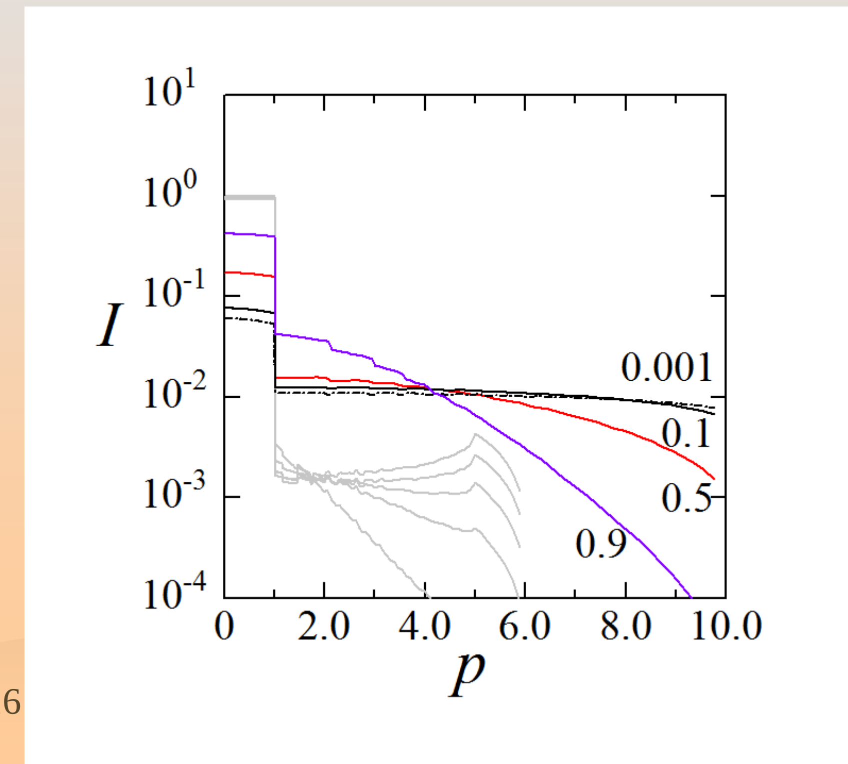


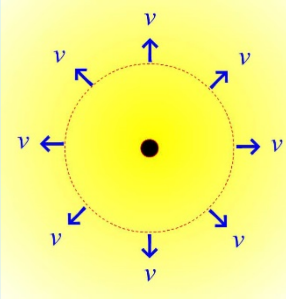
セドフ的流れ　１



- セドフ的な流れの場合（R_{max}=10）
- τ*=3（R_{out}=100R*）

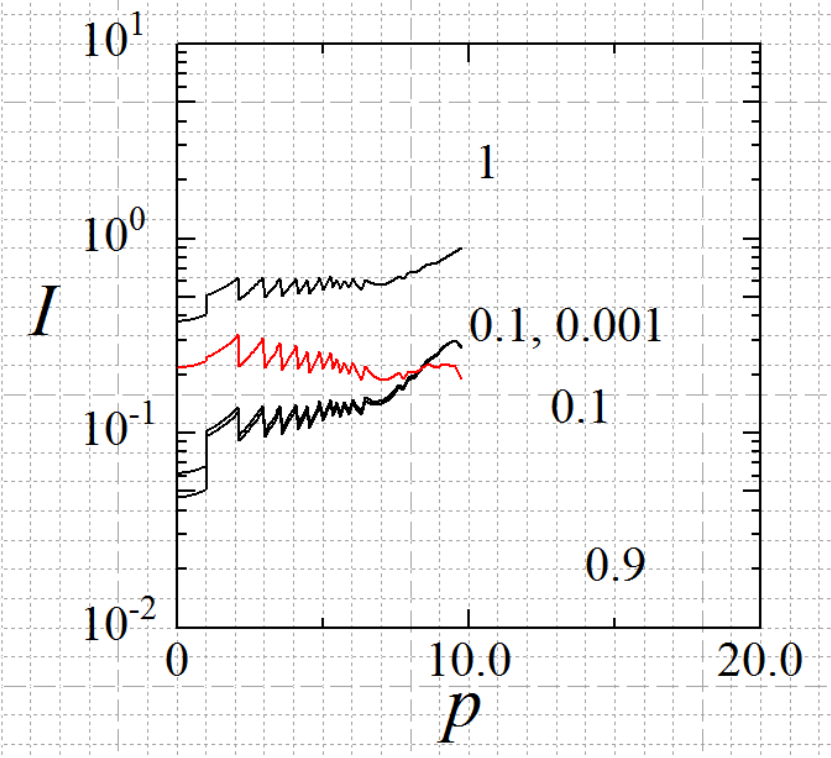






セドフ的流れ　２

- セドフ的な流れの場合（R_{max}=10）
- τ*=3（R_{out}=100R*）

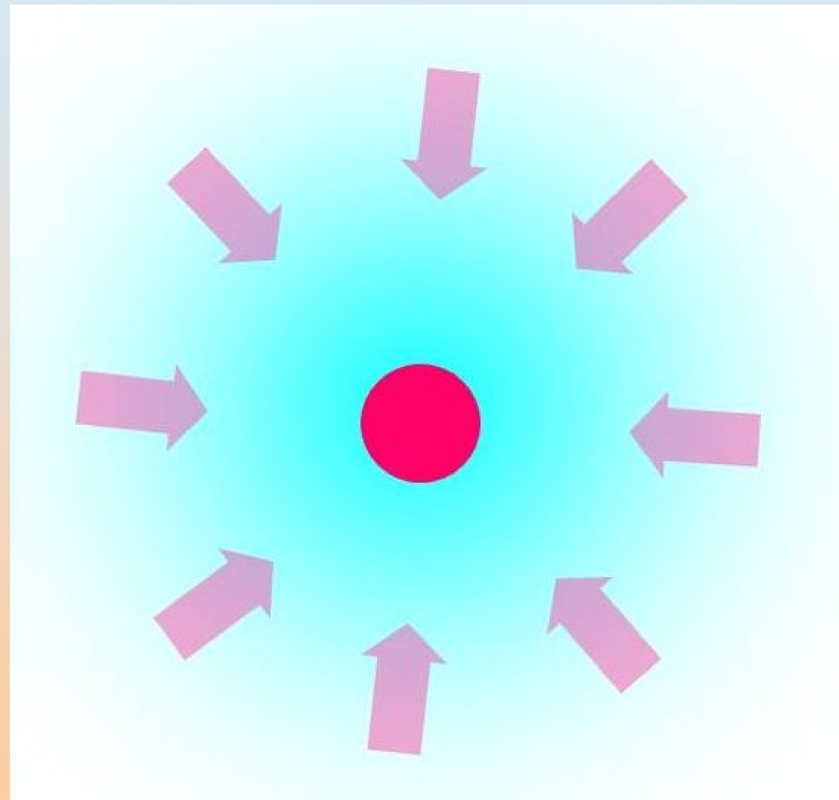


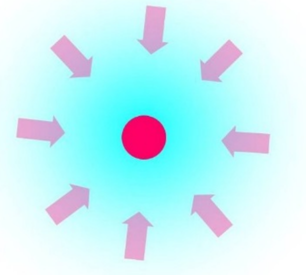
球対称降着　２

- 球状光源：$R*$、$I*=0$
- 自由落下： $\beta=v/c=-1/\sqrt{r}$
- LTE：$S_0=B_0$
- 断熱

- パラメータ

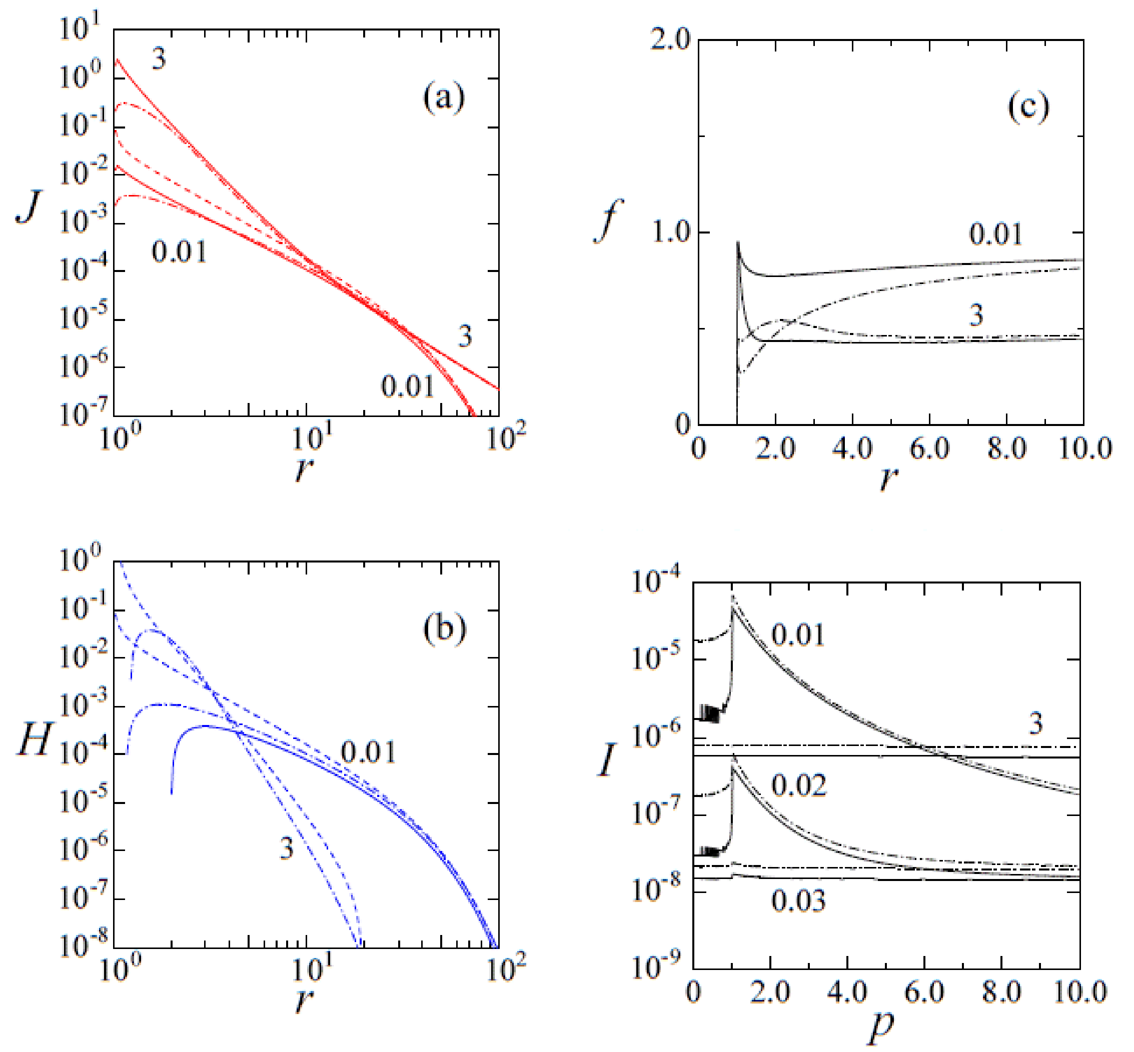
- $\tau*$

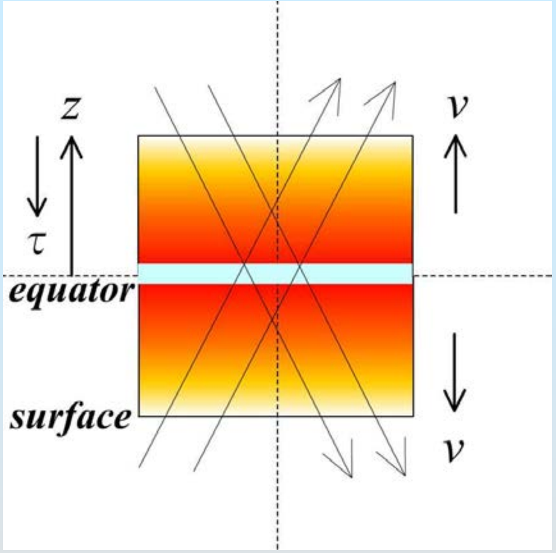




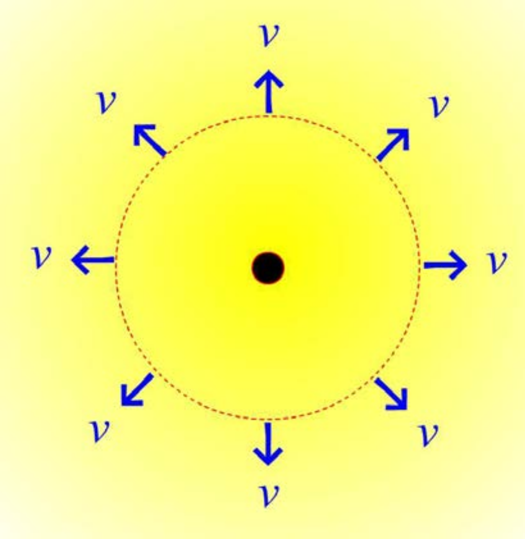
球対称降着　２

- τ^*=3（R_{out}=100





４　次の課題



- 相対論的平行平板流の相対論的形式解の導出
  - 速度場を与えて相対論的輻射輸送を解く
    Fukue 2014
  - 速度場と輻射場を同時に解く
    Fukue 2015
- 相対論的球対称流の相対論的形式解の導出
  - 速度場を与えて相対論的輻射輸送を解く
    Fukue 2017?
  - 速度場と輻射場を同時に解く
    Fukue 2017?

今後の課題

- 精度の問題
  - 光学的厚み:対数メッシュ
  - 角度方向:光行差の問題
- 球対称流
  - 重力場
  - ガス圧
- 振動数依存性、スペクトル