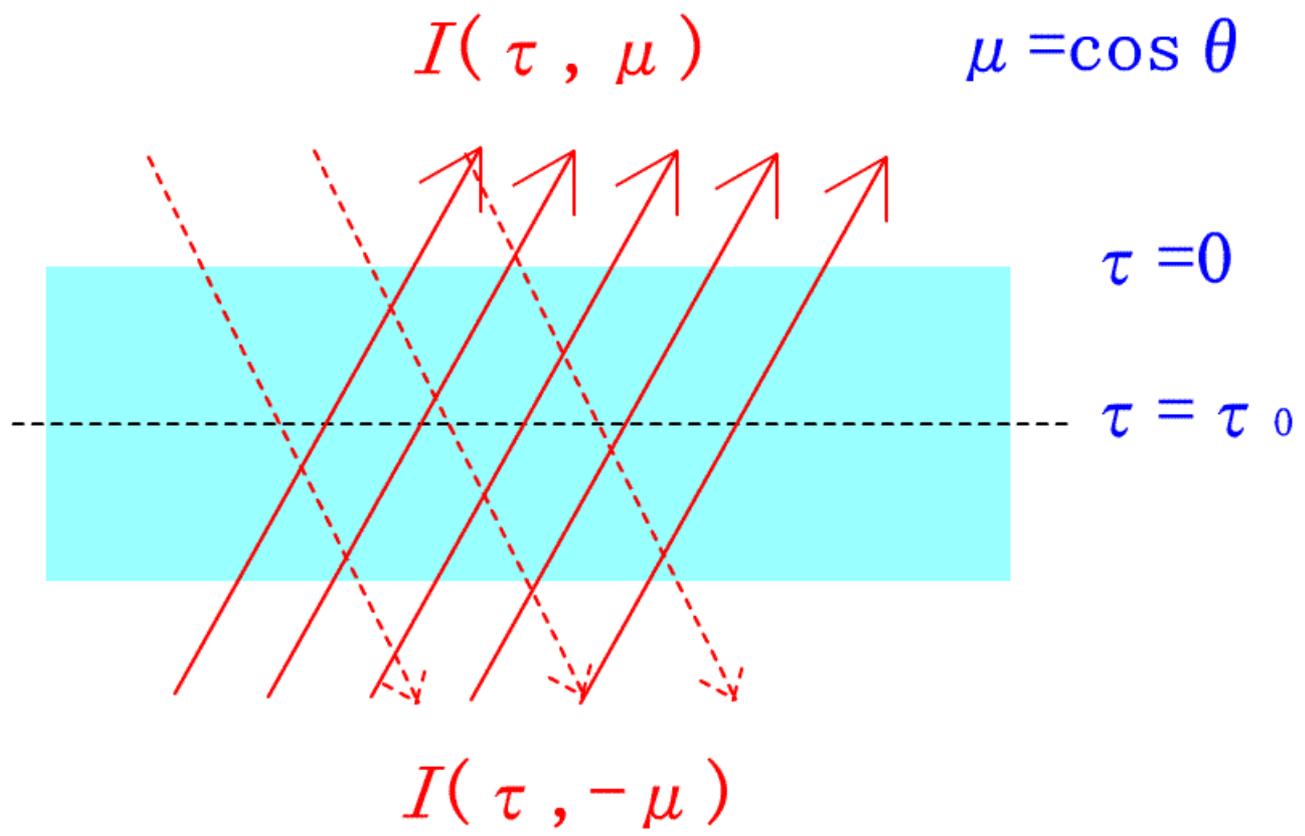
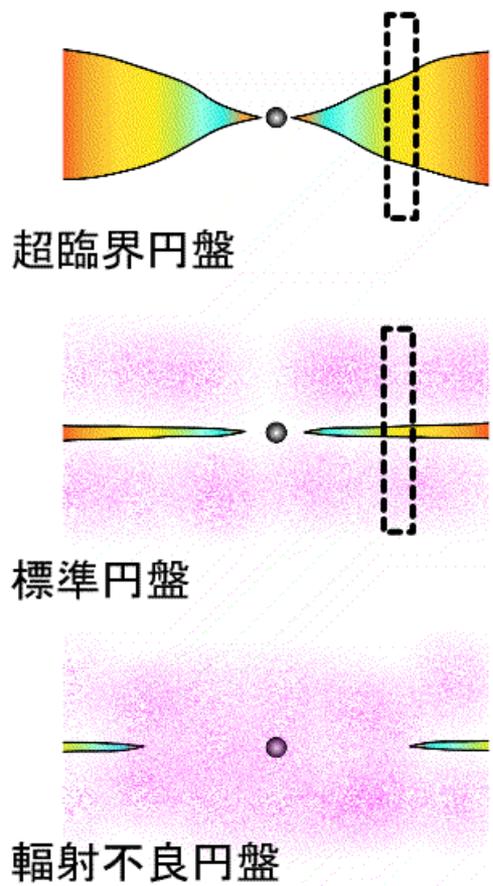




# 降着円盤における輻射輸送 散乱の影響





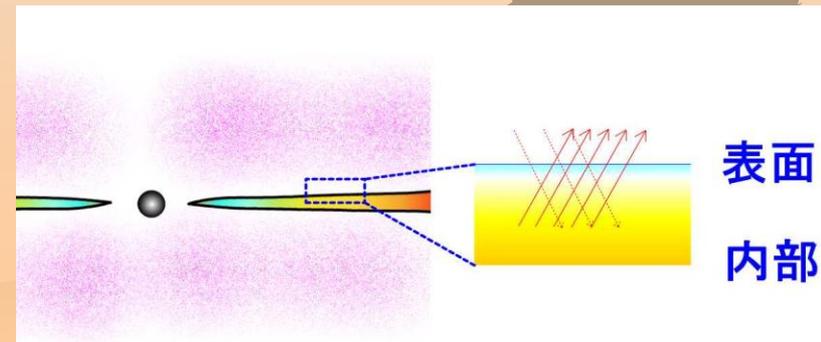
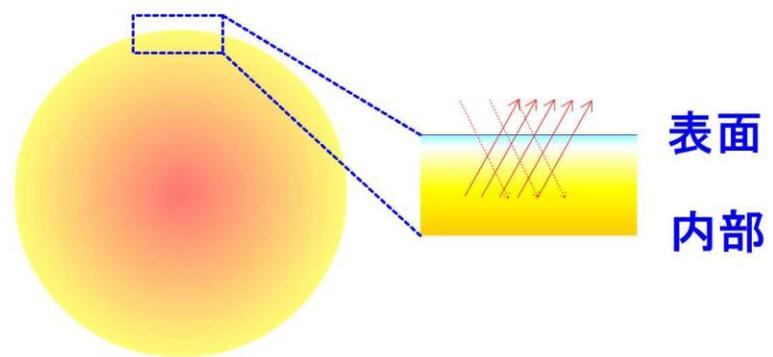
# 1 恒星大気と降着円盤大気

## 恒星大気

- 平行平板近似
- エネルギー源は中心
- 重力場は一定
- 半無限平面
- 散乱はときに重要

## 降着円盤大気

- 平行平板近似
- 加熱源は全体
- 重力場は変化する
- 光学的厚みは有限
- 散乱はたいてい重要





# 降着円盤の輻射輸送 先行研究

Meyer & Meyer-Hofmeister 1982  
 Cannizzo & Wheeler 1984  
 Kriz & Hubeny 1986  
 Shaviv & Wehrse 1986  
 Adam et al. 1988  
 Hubeny 1990  
 Ross et al. 1992  
 Artemova et al. 1996  
 Hubeny & Hubeny 1997, 1998  
 Hubeny et al. 2000, 2001  
 Davis et al. 2005  
 Hui et al. 2005

❁ 散乱の効果  
**Shakura & Sunyaev 1973**  
**Czerny & Elvis 1986**  
**Shaviv & Wehrse 1987**  
**Wandel & Petrosian 1988**  
**Laor & Netzer 1989**  
**Shimura & Takahara 1993**  
**Wang et al. 1999**

- ✓ 拡散近似を使っている
- 無限半平面を仮定している
- 周縁減光効果を考えていない

- ✓ 等温大気を仮定
- ✓ 恒星大気を流用
- →修正黒体輻射MBB





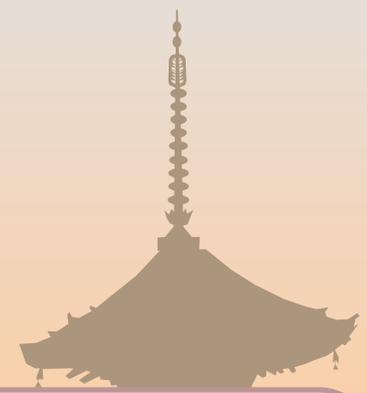
# 降着円盤の輻射輸送 先行研究との比較

Table 1. Analytical Models

Authors	$q^{1)}$	$\tau^{2)}$	$\sigma^{3)}$
Shaviv & Wehrse (1986)	yes	—	—
Czerny & Elvis (1987)	—	—	iso*
Wandel & Petrosian (1988)	—	—	iso
Adams et al. (1988)	yes	—	—
Laor & Netzer (1989)	yes	yes	iso, grad <sup>+</sup>
Hubeny (1990)	yes	yes	—
Wang et al. (1999)	yes	—	grad
Present model	yes	yes	iso, grad

- 1) Heating in the atmosphere
- 2) Finite optical depth
- 3) Scattering effect
- \* Isothermal (modified blackbody)
- + Temperature gradient

**Vertical Radiative Transfer:  
Roles of Scattering Effects**  
by  
Fukue J. 2011, PASJ 63, in press





# 2 非灰色;有限厚+散乱効果

## 基礎方程式1

❁ 輻射輸送方程式

$$\mu \frac{dI_\nu}{dz} = \rho \left[ \frac{j_\nu}{4\pi} - (\kappa_\nu + \sigma_\nu) I_\nu + \sigma_\nu J_\nu \right],$$

❁ 0次のモーメント

$$\frac{dH_\nu}{dz} = \rho \left( \frac{j_\nu}{4\pi} - \kappa_\nu J_\nu \right),$$

❁ 1次のモーメント

$$\frac{dK_\nu}{dz} = -\rho(\kappa_\nu + \sigma_\nu)H_\nu,$$

❁ 静水圧平衡(不使用)

❁ エネルギー式  
(輻射平衡)

$$0 = q_{\text{vis}}^+ - \rho \int (j_\nu - 4\pi\kappa_\nu J_\nu) d\nu,$$

$$\frac{q_{\text{vis}}^+}{4\pi\rho} = \int \kappa_\nu (B_\nu - J_\nu) d\nu$$

$$= \int (\kappa_\nu + \sigma_\nu)(S_\nu - J_\nu) d\nu.$$

非灰色





## 2 非灰色;有限厚+散乱効果 基礎方程式2



- ❁ 輻射輸送方程式
  - ❁ 0次のモーメント
  - ❁ 1次のモーメント
  - ❁ エディントン近似
- 
- ❁ 源泉関数
  - ❁ photon dist. prob.

❁ Rad. diff. equation

2011/9/19



$$\begin{aligned}\mu \frac{dI_\nu}{d\tau_\nu} &= I_\nu - S_\nu, \\ \frac{dH_\nu}{d\tau_\nu} &= J_\nu - S_\nu, \\ \frac{dK_\nu}{d\tau_\nu} &= \frac{1}{3} \frac{dJ_\nu}{d\tau_\nu} = H_\nu.\end{aligned}$$

$$\begin{aligned}S_\nu &= \frac{1}{\kappa_\nu + \sigma_\nu} \frac{j_\nu}{4\pi} + \frac{\sigma_\nu}{\kappa_\nu + \sigma_\nu} J_\nu \\ &= \varepsilon_\nu B_\nu + (1 - \varepsilon_\nu) J_\nu\end{aligned}$$

$$\varepsilon_\nu = \frac{\kappa_\nu}{\kappa_\nu + \sigma_\nu}.$$

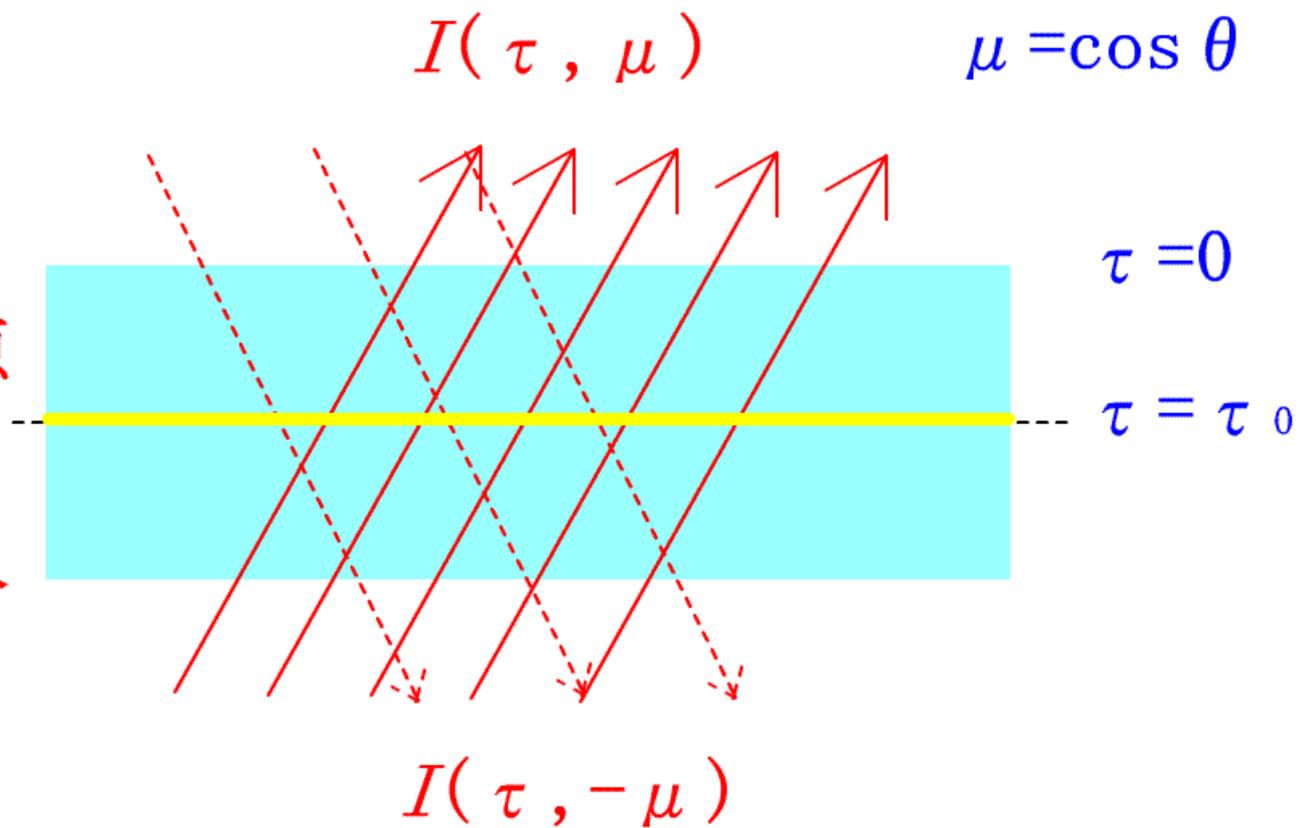
$$\frac{1}{3} \frac{d^2 J_\nu}{d\tau_\nu^2} = J_\nu - S_\nu, = \varepsilon_\nu (J_\nu - B_\nu).$$



# 3 有限厚平行平板の厳密解 解析解 (一様光源、熱源なし)



赤道面上には  
一様等方な光源  
があって、  
円盤大気には  
熱源がない場合





### 3 非灰色;有限厚+散乱効果

解析解 ( $B_\nu(\tau) = B_\nu(0) + b_\nu \tau_\nu$ )



#### ❁ モーメント量

$$\frac{1}{3} \frac{d^2}{d\tau_\nu^2} (J_\nu - B_\nu) = \varepsilon_\nu (J_\nu - B_\nu),$$

$$J_\nu(0) = c_\nu H_\nu(0) \text{ at } \tau_\nu = 0, \quad c_\nu = \sqrt{3},$$

$$J_\nu = B_\nu(0) + b_\nu \tau_\nu - \frac{B_\nu(0) - b_\nu c_\nu / 3}{1 + (c_\nu / 3) \sqrt{3\varepsilon_\nu}} e^{-\sqrt{3\varepsilon_\nu} \tau_\nu},$$

$$H_\nu = \frac{1}{3} b_\nu + \sqrt{\frac{\varepsilon_\nu}{3}} \frac{B_\nu(0) - b_\nu c_\nu / 3}{1 + (c_\nu / 3) \sqrt{3\varepsilon_\nu}} e^{-\sqrt{3\varepsilon_\nu} \tau_\nu},$$

$$S_\nu = B_\nu(0) + b_\nu \tau_\nu - (1 - \varepsilon_\nu) \frac{B_\nu(0) - b_\nu c_\nu / 3}{1 + (c_\nu / 3) \sqrt{3\varepsilon_\nu}} e^{-\sqrt{3\varepsilon_\nu} \tau_\nu}.$$



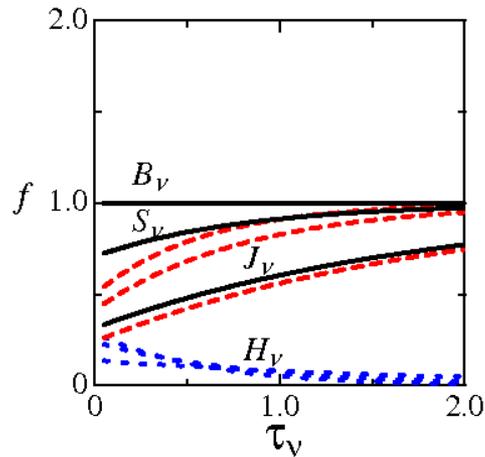


# 3 非灰色;有限厚+散乱效果

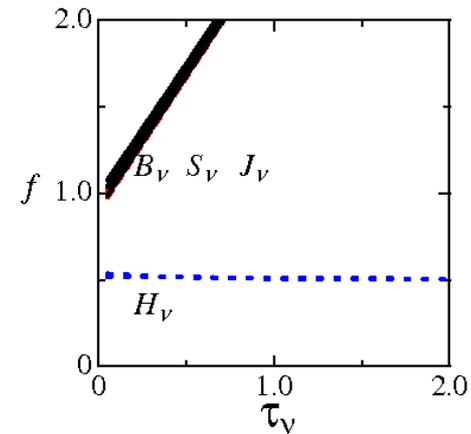
## 解析解 ( $B_\nu(\tau) = B_\nu(0) + b_\nu \tau_\nu$ )

❁ 等温 ( $b_\nu = 0$ )

❁ 温度勾配 ( $b_\nu = 1.5$ )



$\epsilon_\nu =$   
1  
0.5  
0.1



$\epsilon_\nu =$   
1  
0.5  
0.1

图 4.1: Radiative quantities normalized by  $B_\nu(0)$  as a function of the optical depth for an isothermal case ( $b_\nu = 0$ ). Thick dashed curves represent  $J_\nu$ , thick dotted ones  $H_\nu$ , and thick solid ones  $S_\nu$ . The values of the parameter  $\epsilon_\nu$  are 1, 0.5, and 0.1 from top to bottom for each case.

图 4.2: Radiative quantities normalized by  $B_\nu(0)$  as a function of the optical depth for a case with temperature gradient ( $b_\nu/B_\nu(0) = 3/2$ ). Thick dashed curves represent  $J_\nu$ , thick dotted ones  $H_\nu$ , and thick solid ones  $S_\nu$ . Thin solid curves denote the Planck function  $B_\nu$ . The values of the parameter  $\epsilon_\nu$  are 1, 0.5, and 0.1 from top to bottom for each case.





### 3 非灰色;有限厚+散乱效果



解析解 ( $B_\nu(\tau) = B_\nu(0) + b_\nu \tau_\nu$ )

❁ 表面輻射強度

❁ 境界条件

$$I_\nu(\tau_{\nu 0}, \mu) = I_{\nu 0} + I_\nu(\tau_{\nu 0}, -\mu),$$

$$\begin{aligned}
I_\nu(0, \mu) &= I_{\nu 0} e^{-\tau_{\nu 0}/\mu} + B_\nu(0) + b_\nu \mu \\
&\quad - 2b_\nu \mu e^{-\tau_{\nu 0}/\mu} - [B_\nu(0) - b_\nu \mu] e^{-2\tau_{\nu 0}/\mu} \\
&\quad - \frac{1 - \varepsilon_\nu}{1 + \mu\sqrt{3\varepsilon_\nu}} \frac{B_\nu(0) - b_\nu c_\nu/3}{1 + (c_\nu/3)\sqrt{3\varepsilon_\nu}} \left[ 1 - e^{-(1 + \mu\sqrt{3\varepsilon_\nu})\tau_{\nu 0}/\mu} \right] \\
&\quad - \frac{1 - \varepsilon_\nu}{1 - \mu\sqrt{3\varepsilon_\nu}} \frac{B_\nu(0) - b_\nu c_\nu/3}{1 + (c_\nu/3)\sqrt{3\varepsilon_\nu}} \left[ e^{-(1 + \mu\sqrt{3\varepsilon_\nu})\tau_{\nu 0}/\mu} - e^{-2\tau_{\nu 0}/\mu} \right]
\end{aligned}$$





# 3 非灰色;有限厚+散乱效果

## 解析解 ( $B_\nu(\tau) = B_\nu(0) + b_\nu \tau_\nu$ )

❁ 等温 ( $b_\nu = 0$ )

❁ 温度勾配 ( $b_\nu = 1.5$ )

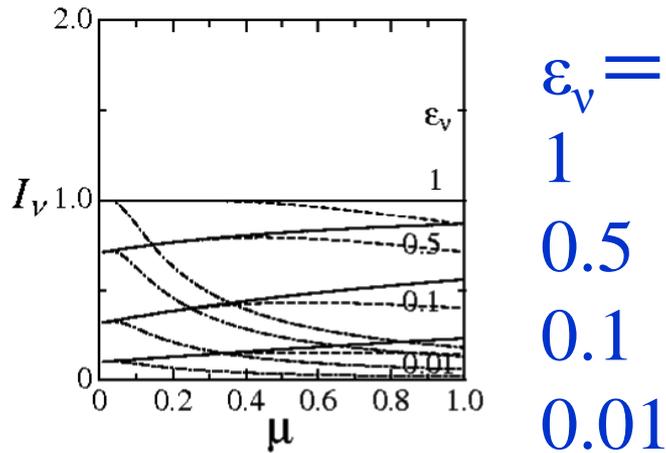


图 4.3: Normalized emergent intensity as a function of  $\mu$  for an isothermal case ( $b_\nu = 0$ ). The values of the disk optical depth  $\tau_{\nu 0}$  are 10 (solid curves), 1 (dashed ones), and 0.1 (chain-dotted ones). The values of the parameter  $\epsilon_\nu$  are 1, 0.5, 0.1, and 0.01 from top to bottom for each case.

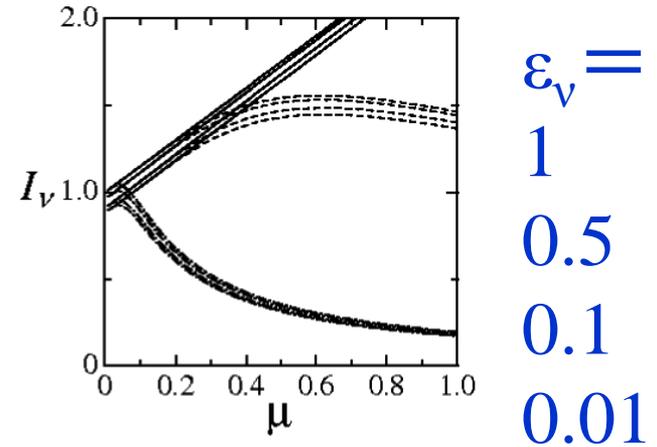


图 4.4: Normalized emergent intensity as a function of  $\mu$  for a case with temperature gradient ( $b_\nu/B_\nu(0) = 3/2$ ). The values of the disk optical depth  $\tau_{\nu 0}$  are 10 (solid curves), 1 (dashed ones), and 0.1 (chain-dotted ones). The values of the parameter  $\epsilon_\nu$  are 1, 0.5, 0.1, and 0.01 from top to bottom for each case.

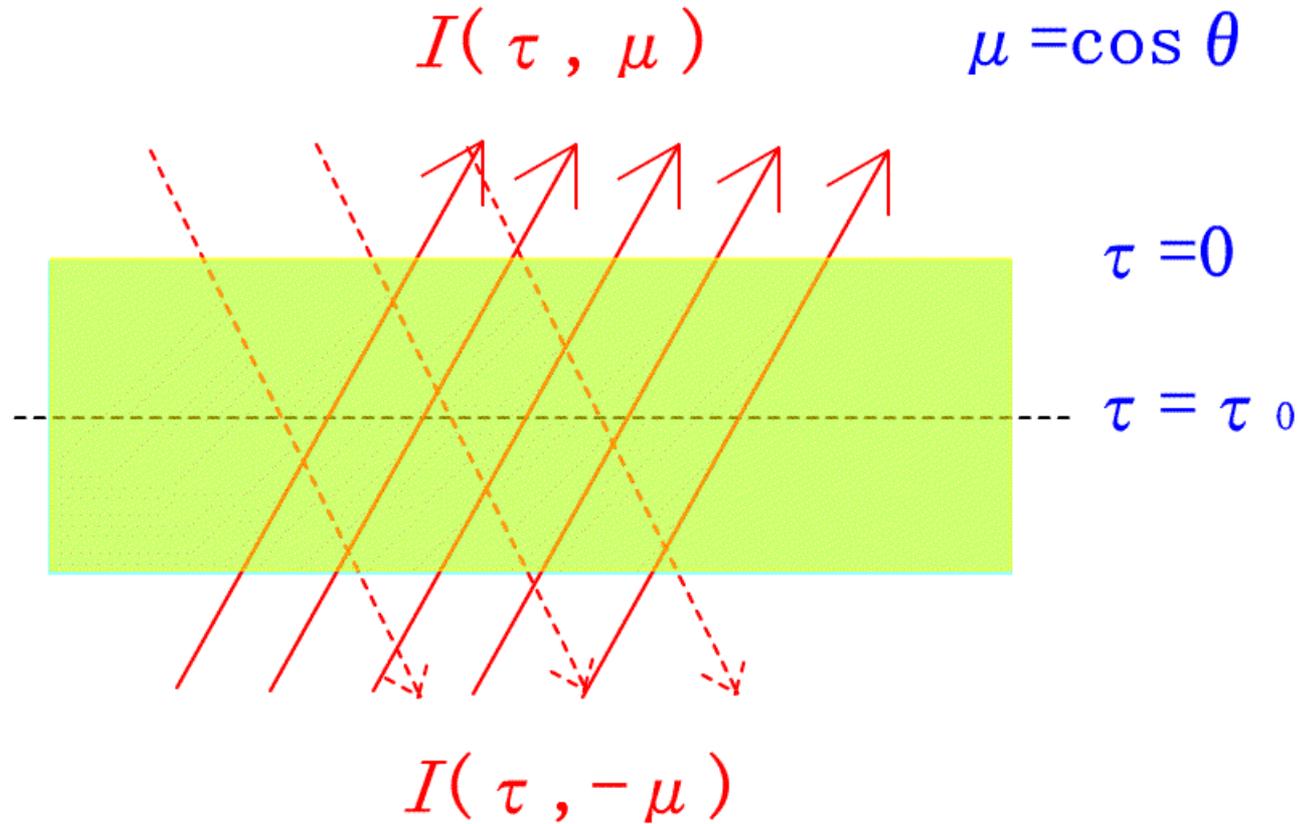




# 4 有限厚平行平板の厳密解 解析解 (熱源あり)



赤道面上には  
光源はなく、  
円盤大気に  
一様な熱源が  
ある場合





# 4 非灰色;有限厚+散乱効果

## 解析解 ( $S_\nu(\tau) - J_\nu(\tau) = q_\nu$ )



### ❁ モーメント量

$$H_\nu = H_\nu(0) \left( 1 - \frac{\tau_\nu}{\tau_{\nu 0}} \right),$$

$$3K_\nu = J_\nu = H_\nu(0) \left( c_\nu + 3\tau_\nu - \frac{3\tau_\nu^2}{2\tau_{\nu 0}} \right),$$

$$S_\nu = H_\nu(0) \left( c_\nu + 3\tau_\nu - \frac{3\tau_\nu^2}{2\tau_{\nu 0}} + \frac{1}{\tau_{\nu 0}} \right),$$

$$B_\nu = H_\nu(0) \left( c_\nu + 3\tau_\nu - \frac{3\tau_\nu^2}{2\tau_{\nu 0}} + \frac{1}{\varepsilon_\nu \tau_{\nu 0}} \right).$$

$$H_\nu(0) = q_\nu \tau_{\nu 0}$$



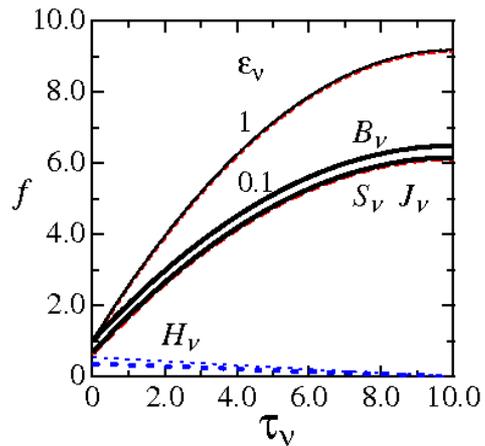


# 4 非灰色;有限厚+散乱効果

## 解析解 ( $S_\nu(\tau) - J_\nu(\tau) = q_\nu$ )

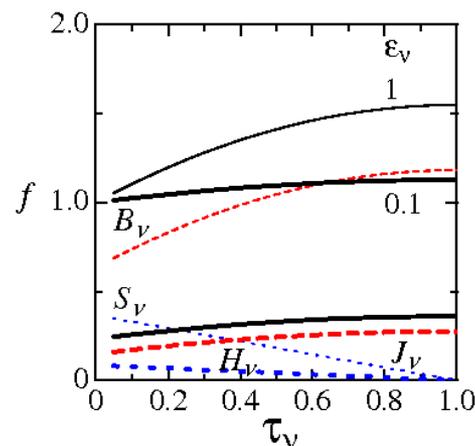
❁ 厚い ( $\tau_{\nu 0} = 10$ )

❁ 薄い ( $\tau_{\nu 0} = 1$ )



$\epsilon_\nu =$   
1  
0.1

図 4.5: Radiative quantities normalized by  $B_\nu(0)$  as a function of the optical depth for large optical depth ( $\tau_{\nu 0} = 10$ ). Dashed curves represent  $J_\nu$ , dotted ones  $H_\nu$ , and solid ones  $B_\nu$  and  $S_\nu$ . The values of the parameter  $\epsilon_\nu$  are 1 (thin curves) and 0.1 (thick ones).



$\epsilon_\nu =$   
1  
0.1

図 4.6: Radiative quantities normalized by  $B_\nu(0)$  as a function of the optical depth for large small depth ( $\tau_{\nu 0} = 1$ ). Dashed curves represent  $J_\nu$ , dotted ones  $H_\nu$ , and solid ones  $B_\nu$  and  $S_\nu$ . The values of the parameter  $\epsilon_\nu$  are 1 (thin curves) and 0.1 (thick ones).





# 4 非灰色;有限厚+散乱效果



解析解 ( $S_\nu(\tau) - J_\nu(\tau) = q_\nu$ )

❁ 表面輻射強度

❁ 境界条件

$$I_\nu(\tau_{\nu 0}, \mu) = I_\nu(\tau_{\nu 0}, -\mu).$$

$$I_\nu(0, \mu) = H_\nu(0) \left[ c_\nu + 3\mu + \frac{1}{\tau_{\nu 0}} (1 - 3\mu^2) - \left( c_\nu - 3\mu + \frac{1}{\tau_{\nu 0}} - \frac{3\mu^2}{\tau_{\nu 0}} \right) e^{-2\tau_{\nu 0}/\mu} \right].$$





# 4 非灰色;有限厚+散乱效果

## 解析解 ( $S_v(\tau) - J_v(\tau) = q_v$ )



- 光学的厚み ( $\tau_{v0}=10, 1, 0.1$ )
- 散乱效果 ( $\varepsilon_v=1, 0.5, 0.1, 0.01$ )

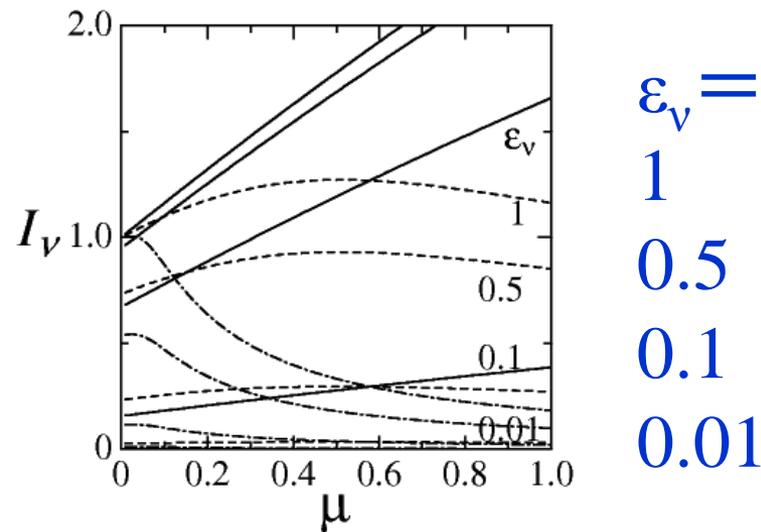


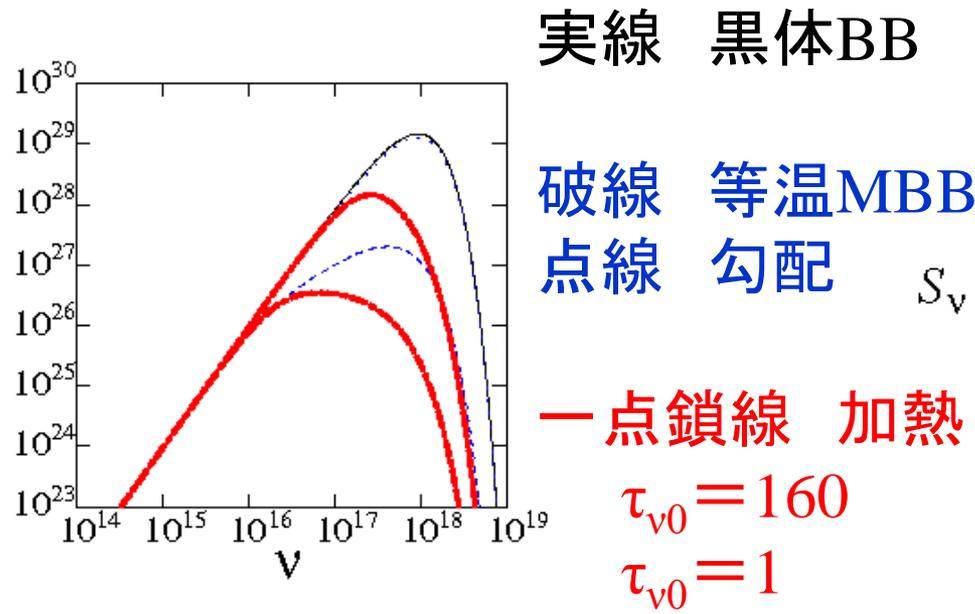
图 4.7: Normalized emergent intensity as a function of  $\mu$ . The values of the disk optical depth  $\tau_{v0}$  are 10 (solid curves), 1 (dashed ones), and 0.1 (chain-dotted ones). The values of the parameter  $\varepsilon_v$  are 1, 0.5, 0.1, and 0.01 from top to bottom for each case.



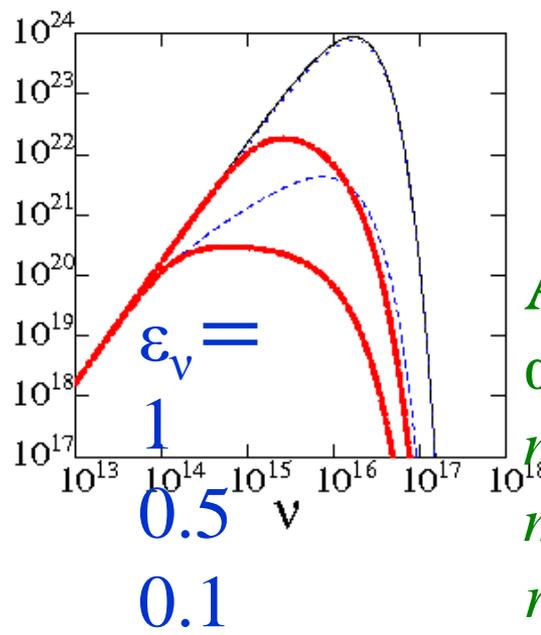


# 5 降着円盤の場合

XRB  $S_\nu$   
 $\alpha = 1$   
 $m = 10$   
 $m = 1$   
 $r = 4$



AGN  $S_\nu$   
 $\alpha = 1$   
 $m = 10^8$   
 $m = 1$   
 $r = 4$   
 $\epsilon_\nu =$   
 1  
 0.5  
 0.1



**Fig. 8.** Surface source function  $S_\nu(0)$  with typical parameters of inner accretion disks of X-ray binaries for several cases. Thin solid curve is the simple blackbody spectrum. Thin dashed and dotted ones are those for no heating cases with isothermal ( $b_\nu = 0$ ) and temperature gradient ( $b_\nu = 3/2$ ), respectively. Thick chain-dotted ones represent those for internal heating cases of  $\tau_{\nu 0} = 160$  (upper) and 1 (lower), respectively. The parameters of standard accretion disks are  $\alpha = 1$ ,  $M/M_\odot = 10$ ,  $\dot{M}/\dot{M}_{\text{crit}} = 1$ , and  $r/r_g = 4$ ,  $\dot{M}_{\text{crit}}$  being the critical accretion rate and  $r_g$  the Schwarzschild radius. In this case the total optical depth is about 160, while the effective optical depth is 0.41.

**Fig. 9.** Surface source function  $S_\nu(0)$  with typical parameters of inner accretion disks of active galactic nuclei for several cases. Thin solid curve is the simple blackbody spectrum. Thin dashed and dotted ones are those for no heating cases with isothermal ( $b_\nu = 0$ ) and temperature gradient ( $b_\nu = 3/2$ ), respectively. Thick chain-dotted ones represent those for internal heating cases of  $\tau_{\nu 0} = 160$  (upper) and 1 (lower), respectively. The parameters of standard accretion disks are  $\alpha = 1$ ,  $M/M_\odot = 10^8$ ,  $\dot{M}/\dot{M}_{\text{crit}} = 1$ , and  $r/r_g = 4$ ,  $\dot{M}_{\text{crit}}$  being the critical accretion rate and  $r_g$  the Schwarzschild radius. In this case the total optical depth is about 160, while the effective optical depth is 0.15.



# まとめと今後

1. 降着円盤のスペクトルは散乱によって修正黒体輻射になると思われているが、これは間違いである。
2. 鉛直方向に温度勾配があれば黒体輻射とあまり変わらない。
3. しかし、光学的厚みの有限性を入れると、合わせ技で、ふたたび黒体輻射からずれる。

- 照射の効果
- 超臨界降着円盤の場合
- 降着円盤風の場合

ブラックホール風など球対称風の場合

