第31章 Relativistic Radiative Transfer Spherical Fixed Frame variables $IEFP$ in physics

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31.1 Relativistic Radiative Transfer Equation and Moment Equations

Let us suppose a spherically moving atmosphere, e.g., a luminous black hole wind. The wind gas moves in the radial direction due to the action of radiation field, while the radiation energy is also transported in the radial direction. For simplicity, in this paper, the radiation field is supposed to be sufficiently intense that both the gravitational field of the central object and the gas pressure can be ignored. We also assume the gray approximation, where the opacities do not depend on the frequency and use the frequency-integrated equations. As for the order of the flow velocity v, we consider the fully special relativistic regime.

The radiative transfer equations are given in several literatures (Chandrasekhar 1960; Mihalas 1970; Rybicki, Lightman 1979; Mihalas, Mihalas 1984; Shu 1991; Kato et al. 1998, 2008; Mihalas, Auer 2001; Peraiah 2002; Castor 2004). The basic equations for relativistic radiation hydrodynamics are given in, e.g., the appendix E of Kato et al. (2008) in general and vertical forms, where the quantities are expressed in the inertial frame. The relativistic radiation hydrodynamic equations in the laboratory frame are also derived and discussed in the seminal paper by Mihalas and Auer (2001) in some details.

In a general form the radiative transfer equation in the inertial (fixed) frame is expressed as

$$\begin{aligned}\frac{1}{c}\frac{\partial I}{\partial t}+(\boldsymbol{l}\cdot\boldsymbol{\nabla})\,I \;=\;& \rho\gamma^{-3}\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)^{-3}\left[\frac{j_0}{4\pi}-\left(\kappa_0^{\rm abs}+\kappa_0^{\rm sca}\right)\gamma^4\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)^4 I+\frac{\kappa_0^{\rm sca}}{4\pi}\frac{3}{4}\gamma^{-2}\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)^{-2}\left\{\gamma^4\left[\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)^2+\left(\frac{v^2}{c^2}-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)^2\right]cE\right.\right.\\ &\left.\left.+2\gamma^2\left(\frac{v^2}{c^2}-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)\boldsymbol{F}\cdot\boldsymbol{l}-2\gamma^4\left[\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)^2+\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)\left(\frac{v^2}{c^2}-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)\right]\frac{\boldsymbol{v}\cdot\boldsymbol{F}}{c}+l_il_jcP^{ij}-2\gamma^2\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)v_il_jP^{ij}+2\gamma^4\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{l}}{c}\right)^2\frac{v_iv_jP^{ij}}{c}\right\}\right]\end{aligned}\tag{31.1}$$

Here, $\boldsymbol{v}$ is the flow velocity, c is the speed of light, and γ $(=1/\sqrt{1-v^2/c^2})$ is the Lorentz factor. In the left-hand side the frequency-integrated specific intensity I and the direction cosine $\boldsymbol{l}$ are quantities measured in the inertial (fixed) frame. In the right-hand side, the mass density ρ, the frequency-integrated mass emissivity j_0, the frequency-integrated mass absorption coefficient $\kappa_0^{\rm abs}$, and the frequency-integrated mass scattering coefficient $\kappa_0^{\rm sca}$ are quantities measured in the comoving (fluid) frame, whereas the frequency-integrated radiation energy density E, the frequency-integrated radiative flux $\boldsymbol{F}$, and the frequency-integrated radiation stress tensor P^{ij} are quantities measured in the inertial (fixed) frame.

In the spherical geometry with radius r and the direction cosine μ $(=\cos\theta)$ at r, the transfer equation is expressed as

$$
\begin{aligned}
\mu\frac{\partial I}{\partial r}+\frac{1-\mu^2}{r}\frac{\partial I}{\partial \mu} \quad = \quad & \rho\frac{1}{\gamma^3(1-\beta\mu)^3}\left[\frac{j_0}{4\pi}-\left(\kappa_0^{\rm abs}+\kappa_0^{\rm sca}\right)\gamma^4(1-\beta\mu)^4 I\right.\\
& +\frac{\kappa_0^{\rm sca}}{4\pi}\frac{3}{4}\gamma^2\left\{\left[1+\frac{(\mu-\beta)^2}{(1-\beta\mu)^2}\beta^2+\frac{(1-\beta^2)^2}{(1-\beta\mu)^2}\frac{1-\mu^2}{2}\right]cE\right.\\
& \left.\left.-\left[1+\frac{(\mu-\beta)^2}{(1-\beta\mu)^2}\right]2F\beta+\left[\beta^2+\frac{(\mu-\beta)^2}{(1-\beta\mu)^2}-\frac{(1-\beta^2)^2}{(1-\beta\mu)^2}\frac{1-\mu^2}{2}\right]cP\right\}\right],
\end{aligned}
\tag{31.2}
$$

where β $(=v/c)$ is the normalized radial speed, and F and P are the radial component of the radiative flux and the radiation stress tensor measured in the inertial frame, respectively.

For matter, the continuity equation, the equation of motion, and the energy equation become, respectively,

$$
4\pi r^2\rho c u \quad = \quad 4\pi r^2\rho\gamma\beta c=\dot{M}\ (=\text{const.}), \tag{31.3}
$$

$$
c^2u\frac{du}{dr} \quad = \quad c^2\gamma^4\beta\frac{d\beta}{dr}=-\frac{d\psi}{dr}-\gamma^2\frac{c^2}{\varepsilon+p}\frac{dp}{dr}+\frac{\rho c^2}{\varepsilon+p}\frac{\kappa_0^{\rm abs}+\kappa_0^{\rm sca}}{c}\gamma^3\left[(1+\beta^2)F-\beta(cE+cP)\right], \tag{31.4}
$$

$$
0 \quad = \quad \frac{q^+}{\rho}-\left(j_0-\kappa_0^{\rm abs}\gamma^2cE-\kappa_0^{\rm abs}u^2cP+2\kappa_0^{\rm abs}\gamma uF\right), \tag{31.5}
$$

where u $(=\gamma\beta)$ is the radial four velocity, $\dot{M}$ the mass-loss rate, ψ the gravitational potential, ε the internal energy per unit proper volume, p the gas pressure, and q^+ the internal heating. In the energy equation (31.5) the advection terms in the left-hand side are dropped under the present cold approximation (radiative equilibrium). We do not use the equation of motion since we will assume that the flow speed is constant.

It should be noted on these assumptions briefly. First, we assume the constant flow speed. The major reason of this assumption is for simplicity to solve moment equations analytically. However, in the terminal stage where the initial acceleration of the black hole wind is finished, the wind speed becomes constant. In such a stage, the gravitational field of the central object is sufficiently weak, the wind gas is enough cold to ignore the pressure gradient force, and the radiative flux is balanced with the radiation drag (i.e., the comoving radiative flux vanishes). As a result, the right-hand side of equation (31.4) is dropped, and the flow velocity becomes constant. In addition, we ignore the advection terms in the energy equation. The major reason of this assumption is also for simplicity. The advection terms are generally important in the moving media. However, in some particular case, where the wind becomes sufficiently cold, or the radiation field is so strong that the radiation energy overcomes the gas internal energy, the advection terms may be safely ignored.

For radiation, the zeroth and first moment equations become, respectively,

$$
\frac{1}{r^2}\frac{d}{dr}\left(r^2F\right) \quad = \quad \rho\gamma\left[j_0-\kappa_0^{\rm abs}cE+\kappa_0^{\rm sca}\gamma^2\beta^2(cE+cP)+\kappa_0^{\rm abs}\beta F-\kappa_0^{\rm sca}(1+\beta^2)\gamma^2\beta F\right], \tag{31.6}
$$

$$
c\frac{dP}{dr} \quad = \quad -\frac{1}{r}(3cP-cE)+\rho\gamma\left[j_0\beta-\kappa_0^{\rm abs}F+\kappa_0^{\rm abs}\beta cP-\kappa_0^{\rm sca}\gamma^2(1+\beta^2)F+\kappa_0^{\rm sca}\gamma^2\beta(cE+cP)\right]. \tag{31.7}
$$

It should be noted that the radiation energy density $E_{\rm co}$, the radiative flux $F_{\rm co}$, and the radiation pressure $P_{\rm co}$ in the comoving frame are respectively expressed as

$$
\begin{aligned}
cE_{\rm co} &= \gamma^2 \left(cE - 2\beta F + \beta^2 cP\right), && (31.8)\\
F_{\rm co} &= \gamma^2 \left[(1+\beta^2)F - \beta(cE + cP)\right], && (31.9)\\
cP_{\rm co} &= \gamma^2 \left(\beta^2 cE - 2\beta F + cP\right). && (31.10)
\end{aligned}
$$

As a closure relation, we assume the Eddington approximation in the comoving frame:

$$
f(\tau, \beta) \equiv \frac{P_{\rm co}}{E_{\rm co}}, \qquad (31.11)
$$

and this relation is expressed by the quantities in the inertial frame as

$$
(1 - f\beta^2)cP = (f - \beta^2)cE + 2\beta(1-f)F, \qquad (31.12)
$$

where $f(\tau, \beta)$ is the variable Eddington factor, which generally depends on the velocity and its gradient as well as the optical depth (Fukue 2006b, 2008b, 2008d, 2009).

Eliminating j_0 using the energy equation (31.5), moment equations (31.6) and (31.7) become, repectively,

$$
\begin{aligned}
\frac{1}{r^2}\frac{d}{dr}\left(r^2 F\right) &= \gamma q^+ - \left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\rho\gamma^3\beta\left[(1+\beta^2)F - \beta(cE + cP)\right], && (31.13)\\
c\frac{dP}{dr} &= -\frac{1}{r}\left(3cP - cE\right) + \gamma\beta q^+ - \left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\rho\gamma^3\left[(1+\beta^2)F - \beta(cE + cP)\right]. && (31.14)
\end{aligned}
$$

In the plane-parallel case (Fukue 2008c), the basic equations are expressed using the optical depth defined by

$$
d\tau = -\left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\rho dr. \qquad (31.15)
$$

In the present spherical case, however, there remains a variable r due to the existence of the curvature term. Hence, we use both variables, r and τ.

Finally, using the closure relation (31.12), and dropping the internal heating term, moment equations (31.13) and (31.14) are expressed, respectively, as

$$
\begin{aligned}
\frac{d}{dr}\left(4\pi r^2 F\right) &= -4\pi r^2\left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\rho\gamma\beta F_{\rm co}, && (31.16)\\
\frac{d}{dr}\left(cP\right) &= -\frac{1}{r}\left(3cP - cE\right) - \left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\rho\gamma F_{\rm co}, && (31.17)
\end{aligned}
$$

where

$$
\begin{aligned}
F_{\rm co} &= \frac{f+\beta^2}{f-\beta^2}F - \frac{(1+f)(1-\beta^2)\beta}{f-\beta^2}cP, && (31.18)\\
3cP - cE &= \frac{2(1-f)\beta}{f-\beta^2}F - \frac{1-3f+(3-f)\beta^2}{f-\beta^2}cP. && (31.19)
\end{aligned}
$$

31.2 Solutions of Moment Equations

In this paper, the flow speed is assumed to be constant, and we seek analytical solutions of moment equations (31.16) and (31.17) under appropriate situations and boundary conditions. In the following two subsections we show analytical solutions, which are reduced to those of the diffusion and streaming limit, respectively, in the non-relativistic regime.

31.2.1 Solutions in the Diffusion Limit

Let us first consider the case, where the comoving luminosity is constant. Namely, we assume that

$$L_{\rm co} \equiv 4\pi r^2 F_{\rm co} = {\rm const.} \tag{31.20}$$

In this case, introducing the optical depth (31.15), the zeroth moment equation (31.16) is rewritten as

$$\frac{d}{d\tau}\left(4\pi r^2 F\right) = \gamma\beta L_{\rm co}, \tag{31.21}$$

which is easily integrated to give the luminosity in the inertial frame:

$$L \equiv 4\pi r^2 F = L_\infty + L_{\rm co}\gamma\beta\tau, \tag{31.22}$$

where L_∞ is the luminosity at $\tau = 0$ (i.e., $r = \infty$).

As is well-known, in the non-relativistic regime of the spherically symmetric radiative transfer, the luminosity is constant unless there is no heating or cooling. In the present relativistic regime, on the other hand, the inertial luminosity increases as the optical depth increases, even if the comoving luminosity is constant. In other words, the inertial luminosity decreases as the optical depth decreases (or the radius increases). This is just the result that radiation gives its momentum to matter through the relativistic interaction on the right-hand side of equation (31.16).

Using solution (31.22), the first moment equation (31.17) become

$$\frac{d}{dr}\left(cP\right) = \frac{a_1}{r}cP - \frac{a_2}{r}\frac{L_\infty + L_{\rm co}\gamma\beta\tau}{4\pi r^2} - \rho\left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\gamma\frac{L_{\rm co}}{4\pi r^2}, \tag{31.23}$$

where

$$a_1 \equiv \frac{1 - 3f + (3-f)\beta^2}{f - \beta^2}, \tag{31.24}$$

$$a_2 \equiv \frac{2(1-f)\beta}{f - \beta^2}. \tag{31.25}$$

This equation (31.23) is rearranged as

$$\frac{d}{dr}\left(\frac{cP}{r^{a_1}}\right) = -\frac{a_2 L_\infty}{4\pi}\frac{1}{r^{a_1+3}} - \frac{a_2 L_{\rm co}\gamma\beta}{4\pi}\frac{\tau}{r^{a_1+3}} - \rho\left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\gamma\frac{L_{\rm co}}{4\pi r^{a_1+2}}. \tag{31.26}$$

Here, we assume the power-law type relation between the optical depth and radius in the diffusion limit like

$$\tau = \tau_0\left(\frac{r_0}{r}\right)^a, \tag{31.27}$$

where τ_0 is the optical depth at some reference radius r_0, and a is an arbitrary parameter (cf. Kosirev 1934; Chandrasekhar 1934). Then, using the definition of the optical depth (31.15), we have

$$\rho\left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right) = \frac{a\tau_0}{r_0}\left(\frac{r_0}{r}\right)^{a+1}. \tag{31.28}$$

In the present case of the constant flow speed, from the continuity equation (31.3), the mass density varies as $\rho \propto r^{-2}$. Hence, as long as the opacity is constant such as the Thomson scattering one, the parameter a is fixed to be unity.

When the optical depth is expressed as (31.27), equation (31.26) can be integrated to give a solution:

$$cP = C_0 r^{a_1} + \frac{a_2}{a_1+2}\frac{L_\infty}{4\pi r^2} + \frac{a_2\beta + a}{a_1 + a + 2}\frac{L_{\rm co}}{4\pi r^2}\gamma\tau, \tag{31.29}$$

where C_0 is an integration constant. In the limit of $\beta = 0$ and $f = 1/3$, this equation (31.29) reduces to the solution in the non-relativistic regime, if $C_0 = cP_\infty$, P_∞ being the radiation pressure at $\tau = 0$.

In the present relativistic regime, in order for the pressure not to diverge at $r \to \infty$, the coefficient a_1 should be zero. By this restriction and equation

(31.24), the following condition should be satisfied,

$$f = \frac{1+3\beta^2}{3+\beta^2}. \tag{31.30}$$

This is just the generalized variable Eddington factor derived in Fukue (2009).

Using these relations and conditions, the spherical pressure in the diffusion limit finally becomes

$$Q \equiv 4\pi r^2 cP = 4\pi r^2 cP_\infty + \frac{2\beta^2}{1+\beta^2}L_\infty + \frac{a+(4\beta^3)/(1+\beta^2)}{a+2}L_{\rm co}\gamma\tau. \tag{31.31}$$

It is noted that, in the non-relativistic regime, the relevant solution is $Q = 4\pi r^2 cP_\infty + [a/(a+2)]L_{\rm co}\tau$.

For the convenience of readers, typical examples of the solutions in the diffusion limit are depicted in figure 1 for various values of the flow speed. In figure 1a, the inertial luminosity L normalized by L_∞ is shown as a function of τ. The values of β are 0, 0.2, 0.5, 0.9 from bottom to top for each curve. The values of $L_{\rm co}/L_\infty = 1$ is set to be unity. As is seen from equation (31.22) and figure 1a, in the non-relativistic limit the inertial luminosity is constant, whereas it is proportional to τ for $\beta > 0$.

In figure 1b, the modified part of the spherical pressure $(Q - 4\pi r^2 cP_\infty)$ normalized by L_∞ is shown as a function of τ. The values of β are 0, 0.2, 0.5, 0.9 from bottom to top for each curve. The values of parameters are set as $a = 1$ and $L_{\rm co}/L_\infty = 1$. As is seen from equation (31.31) and figure 1b, in the relativistic regime the spherical pressure is rather enhanced compared with that in the non-relativstic regime. However, the qualitative behavior is similar to the non-relativistic regime.

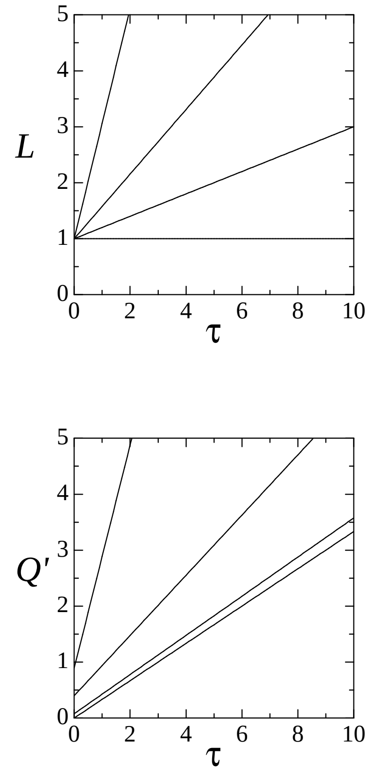

図 31.1: Diffusion-limit solutions for the relativistic spherical flows: (a) Normalized luminosity L/L_∞, (b) Normalized spherical pressure $(Q - 4\pi r^2 cP_\infty)/L_\infty$. The values of β are 0, 0.2, 0.5, 0.9 from bottom to top for each curve. The values of parameters are set as $a = 1$ and $L_{\rm co}/L_\infty = 1$.

31.2.2 Solutions in the Streaming Limit

Next, we consider the case, where the comoving luminosity is not constant. If the velocity is constant and the variable Eddington factor does not depend on radius, moment equations (31.16) and (31.17) are generally written as

$$\frac{1}{4\pi r^2}\frac{d}{dr}\left(4\pi r^2 F\right) = -\left(\kappa_0^{\rm abs} + \kappa_0^{\rm sca}\right)\rho\gamma\beta F_{\rm co}, \tag{31.32}$$

$$\frac{d}{dr}(\beta cP) = \frac{a_1}{r}\beta cP - \frac{a_2}{r}\beta F - \left(\kappa_0^{\rm abs}+\kappa_0^{\rm sca}\right)\rho\gamma\beta F_{\rm co}. \quad (31.33)$$

Eliminating $F_{\rm co}$ from these equations (31.32) and (31.33), we have

$$\frac{d}{dr}\left(r^{2-a_2\beta}F\right) = r^{2+a_1-a_2\beta}\frac{d}{dr}\left(r^{-a_1}\beta cP\right). \quad (31.34)$$

Here, from equations (31.24) and (31.25),

$$2+a_1-a_2\beta = \frac{(1-f)(1-\beta^2)}{f-\beta^2}. \quad (31.35)$$

Hence, if $f=1$ (i.e., the streaming limit; $a_1=-2$ and $a_2=0$), $2+a_1-a_2\beta=0$, and equation (31.34) can be integrated to be

$$r^2F = r^2\beta cP + F_1(\beta), \quad (31.36)$$

where $F_1(\beta)$ is an arbitrary function of β.

Alternatively, in this streaming limit of $f=1$ ($a_1=-2$ and $a_2=0$), moment equations (31.32) and (31.33) are rewritten as

$$\frac{dL}{d\tau} = \gamma\beta L_{\rm co}, \quad (31.37)$$

$$\frac{dQ}{d\tau} = \gamma L_{\rm co}, \quad (31.38)$$

which has an integration of

$$L = \beta Q + L_1(\beta), \quad (31.39)$$

where $L_1(\beta) = 4\pi F_1(\beta)$.

Inserting this relation (31.39) into equations (31.37) and (31.38), we can integrate equations (31.37) and (31.38) to give the following forms:

$$L = \gamma^2 L_1(\beta)\left[(1+f) - C_1(\beta)(f-\beta^2)e^{-\frac{\beta}{\gamma(f-\beta^2)}\tau}\right] = \gamma^2 L_1\left[2 - C_1(1-\beta^2)e^{-\gamma\beta\tau}\right], \quad (31.40)$$

$$Q = \frac{\gamma^2 L_1(\beta)}{\beta}\left[(f+\beta^2) - C_1(\beta)(f-\beta^2)e^{-\frac{\beta}{\gamma(f-\beta^2)}\tau}\right] = \frac{\gamma^2 L_1}{\beta}\left[(1+\beta^2) - C_1(1-\beta^2)e^{-\gamma\beta\tau}\right], \quad (31.41)$$

where $C_1(\beta)$ is an arbitrary function of β.

These equations (31.40) and (31.41) are general solutions of relativistic moment equations for spherical flows at a constant speed in the streaming limit (in the streaming limit $E=P$ and $E_{\rm co}=P_{\rm co}$). It should be noted that these solutions (31.40) and (31.41) have quite similar forms to those of the plane-parallel case (Fukue 2008c), if we replace L and Q in the present spherical case by F and cP in the plane-parallel case, respectively.

We now impose the boundary condition at $\tau=0$. In the streaming limit the appropriate boundary condition at $\tau=0$ is

$$L = Q = L_\infty(\text{const.}), \quad (31.42)$$

where L_∞ is the luminosity at infinity. This boundary condition is different from that for the plane-parallel case, and therefore, the following specitial solutions are also different from those for the plane-parallel case.

Using this boundary condition (31.42), we can determine the arbitrary functions L_1 and C_1 as

$$L_1 = (1-\beta)L_\infty, \quad (31.43)$$

$$C_1 = \frac{f-\beta}{f-\beta^2} = \frac{1}{1+\beta}. \quad (31.44)$$

Finally, the solutions of moment equations for relativistic spherical flows at a constant speed in the streaming limit are explicitly written as

$$\frac{L}{L_\infty} = \frac{1}{1+\beta}\left[2-(1-\beta)e^{-\gamma\beta\tau}\right], \quad (31.45)$$

$$\frac{Q}{L_\infty} = \frac{1}{\beta(1+\beta)}\left[(1+\beta^2)-(1-\beta)e^{-\gamma\beta\tau}\right]. \quad (31.46)$$

In the limit of small β, these solutions approach

$$\frac{L}{L_\infty} \sim 1+\frac{\beta}{1+\beta}\tau, \quad (31.47)$$

$$\frac{Q}{L_\infty} \sim 1+\frac{1-\beta}{1+\beta}\tau. \quad (31.48)$$

These are just the Milne-Eddington type solutions for a static spherical atmosphere. Hence, the present solutions (31.45) and (31.46) are some generalized

Milne-Eddington ones extended to the case for the relativistic spherical flow in the streaming limit.

The solutions of moment equations in the comoving frame are also explicitly written as

$$\frac{L_{\rm co}}{L_\infty} = \frac{1-\beta}{1+\beta}e^{-\gamma\beta\tau}, \tag{31.49}$$

$$\frac{Q_{\rm co}}{L_\infty} = \frac{1-\beta}{\beta(1+\beta)}\left[(1+\beta)-e^{-\gamma\beta\tau}\right]. \tag{31.50}$$

For the convenience of readers, typical examples of the solutions in the streaming limit are depicted in figure 2 for various values of the flow speed. In figure 2a, the inertial luminosity L normalized by L_∞ is shown as a function of τ by solid curves, whereas the comoving one $L_{\rm co}$ by dashed ones. The values of β are 0 to 0.9 in steps of 0.1 from bottom to top (solid curves) and from top to bottom (dashed ones) for each curve. In figure 2b, the inertial spherical pressure Q normalized by L_∞ is shown as a function of τ by solid curves, whereas the comoving one $Q_{\rm co}$ by dashed ones. The values of β are 0 to 0.9 in steps of 0.1 from top to bottom for each curve.

As is seen from equations (31.45) and (31.46) and figure 2, the luminosity in the inetial frame, which is constant in the non-relativistic regime, increases as the flow velocity increases. On the other hand, the spherical pressure in the inertial frame, which is proportional to the optical depth in the non-relativistic regime, decreases as the flow velocity increases. Such a relativistic enhancement is similar to the plane-parallel case.

On the other hand, as is seen from equations (31.49) and (31.50) and figure 2, both the comoving luminosity and spherical pressure are reduced in the relativistic regime. This is because in the streaming limit radiation and matter move together as the flow velocity increases.

We finally emphasize that, similar to the plane-parallel case, there appear exponential terms in these solutions.

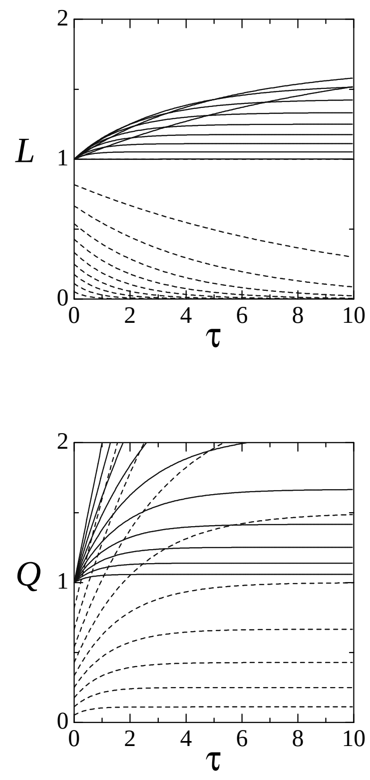

図 31.2: Streaming-limit solutions for the relativistic spherical flows: (a) Normalized luminosities L/L_∞ in the inertial frame (solid curves) and $L_{\rm co}/L_\infty$ in the comoving frame (dashed ones), The values of β are 0 to 0.9 in steps of 0.1 from bottom to top (solid curves) and from top to bottom (dashed ones) for each curve. (b) Normalized spherical pressures Q/L_∞ in the inertial frame (solid curves) and $Q_{\rm co}/\infty$ in the comoving frame (dashed ones). The values of β are 0 to 0.9 in steps of 0.1 from top to bottom for each curve.